Chapter 10, Conceptual Questions

10.2. Kinetic energy depends on speed. Potential energy depends on position.

10.3. No, kinetic energy can never be negative. Kinetic energy is energy of motion. Motion may stop, but it can't be negative. Speed has no direction and cannot be negative. Yes, gravitational potential energy can be negative. Potential energy depends upon position, which can be positive or negative.

10.4. We must calculate the new kinetic energy and compare it to the original value. Originally, $K = \frac{1}{2}mv^2.$ With a velocity of $3v$, $K' = \frac{1}{2}m(3v)^2 = 9\left(\frac{1}{2}mv^2\right) = 9K.$ The kinetic energy increases by a factor of 9.

10.5. We have

$$K_A = 8K_B$$
$$\frac{1}{2}m_A v_A^2 = 8\left(\frac{1}{2}m_B v_B^2\right)$$

Since $m_A = \frac{1}{2}m_B$,

$$\frac{1}{2}\left(\frac{1}{2}m_B\right)v_A^2 = 8\left(\frac{1}{2}m_B v_B^2\right)$$
$$\Rightarrow \frac{v_A}{v_B} = 4$$

10.6. Conservation of energy tells us that $U_i = K_f$, since the car starts at rest. Originally, this means that in rolling down a track of height h,

$$mgh = \frac{1}{2}mv_0^2$$

To go twice as fast at the bottom, we must find the height h' such that

$$mgh' = \frac{1}{2}m(2v_0)^2$$
$$\Rightarrow mgh' = 4\left(\frac{1}{2}mv_0^2\right).$$

So $h' = 4h.$ You must increase the track height by a factor of 4.

10.7. $v_a = v_b = v_c.$ They each start with the same kinetic energy and they each have the same change in potential energy, so they end with the same kinetic energy and, thus, the same speed.

10.8. $v_a = v_b = v_c.$ The balls start off with the same kinetic energy and have the same change in potential energy, so their final kinetic energy is the same.

10.9. **(a)** We identify the equilibrium position $s_e = 10$ cm. At $s = 11$ cm, $\Delta s = s - s_e = 11 \text{ cm} - 10 \text{ cm} = 1 \text{ cm}$, and $F_{sp} = F = -k\Delta s = -k(1 \text{ cm}).$ To get $F_{sp} = 3F$, we must have $3F = -k(\Delta s)'$, which means $(\Delta s)' = 3$ cm. So the spring must have length $10 \text{ cm} + 3 \text{ cm} = 13$ cm.
(b) Note that the direction of the force is reversed when the spring is compressed.

To get $F_{sp} = -2F$, we must have $-2F = -k(\Delta s)'$, which means $(\Delta s)' = -2$ cm. So the spring must have length $10 \text{ cm} - 2 \text{ cm} = 8$ cm.

10.10. Note that Carlos takes the place of the wall, and that the force on the spring is still 200 N. The spring still stretches 20 cm, but now that distance is split between Carlos's and Bob's motion. Bob is pulling to the right, so Carlos pulls to the left and moves half the stretch length, 10 cm.

10.11. $(U_s)_d > (U_s)_c > (U_s)_b = (U_s)_a$. $U_s = \frac{1}{2}k(\Delta s)^2$. Increasing the stretch by a factor of 2 increases the stored energy by a factor of 4. Doubling k doubles the stored energy.

10.12. The original spring stores energy $U = \frac{1}{2}k(1.0\text{ cm})^2$. For a spring with spring constant $k' = 2k$,

$$U' = \frac{1}{2}k'(\Delta s)^2 = \frac{1}{2}(2k)(\Delta s)^2$$

If $U' = U$, then

$$\frac{1}{2}k(1.0\text{ cm})^2 = \frac{1}{2}(2k)(\Delta s)^2$$
$$\Rightarrow \Delta s = \frac{1}{\sqrt{2}}\text{ cm} = 0.71\text{ cm}$$

10.13. Energy conservation tells us that the initial potential energy stored in the spring is equal to the final kinetic energy of the ball.

$$\frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_0^2$$

When the spring is compressed twice as far,

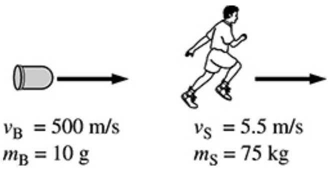
$$\frac{1}{2}k(2\Delta s)^2 = \frac{1}{2}m(2v_0)^2$$

So the ball speed increases by a factor of 2.

10.15. The problem can be divided into three parts: (1) from when the first ball is released and to just before it hits the stationary ball, (2) the two balls collide, and (3) the two balls swing up together just after the collision to their highest point. Energy is conserved in parts (1) and (3) as the balls swing like pendulums, but during the collision in part (2) momentum is conserved but energy is not. So both energy and momentum conservation are each separately used as you work through each part of the problem.

Chapter 10, Exercises and Problems

10.1. Model: We will use the particle model for the bullet (B) and the running student (S).
Visualize:



Solve: For the bullet,

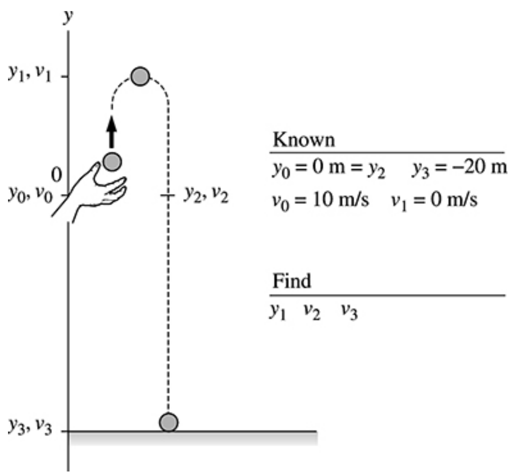
$$K_B = \frac{1}{2}m_B v_B^2 = \frac{1}{2}(0.010 \text{ kg})(500 \text{ m/s})^2 = 1250 \text{ J}$$

For the running student,

$$K_S = \frac{1}{2}m_S v_S^2 = \frac{1}{2}(75 \text{ kg})(5.5 \text{ m/s})^2 = 206 \text{ J}$$

Thus, the bullet has the larger kinetic energy.
Assess: Kinetic energy depends not only on mass but also on the square of the velocity. The above calculation shows this dependence. Although the mass of the bullet is 7500 times smaller than the mass of the student, its speed is more than 90 times larger.

10.5. Model: This is a case of free fall, so the sum of the kinetic and gravitational potential energy does not change as the ball rises and falls.
Visualize:



The figure shows a ball's before-and-after pictorial representation for the three situations in parts (a), (b) and (c).
Solve: The quantity $K + U_g$ is the same during free fall: $K_f + U_{gf} = K_i + U_{gi}$. We have

(a) $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$

$$\Rightarrow y_1 = \left(v_0^2 - v_1^2\right)\big/2g = [(10 \text{ m/s})^2 - (0 \text{ m/s})^2]/(2\times 9.8 \text{ m/s}^2) = 5.10 \text{ m}$$

5.1 m is therefore the maximum height of the ball above the window. This is 25.1 m above the ground.

(b) $\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_0^2 + mgy_0$

Since $y_2 = y_0 = 0,$ we get for the magnitudes $v_2 = v_0 = 10 \text{ m/s}.$

(c) $\frac{1}{2}mv_3^2 + mgy_3 = \frac{1}{2}mv_0^2 + mgy_0 \Rightarrow v_3^2 + 2gy_3 = v_0^2 + 2gy_0 \Rightarrow v_3^2 = v_0^2 + 2g(y_0 - y_3)$

$$\Rightarrow v_3^2 = (10 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)[0 \text{ m} - (-20 \text{ m})] = 492 \text{ m}^2/\text{s}^2$$

This means the magnitude of v_3 is equal to 22 m/s.

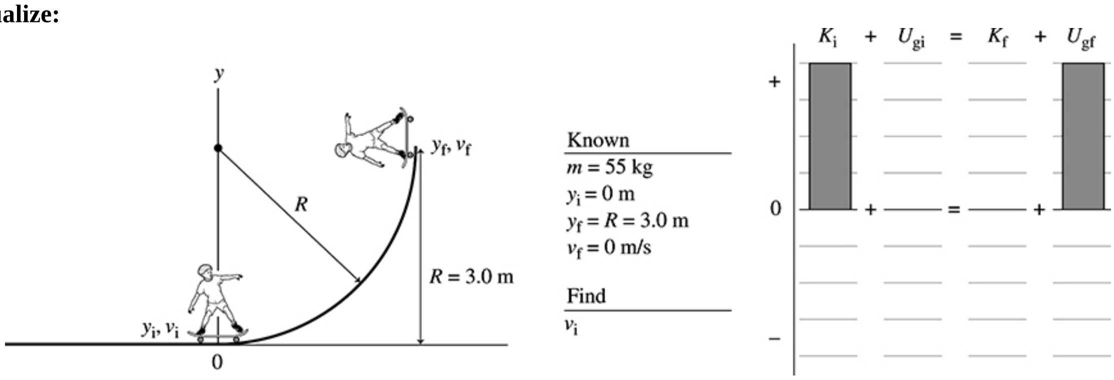
Assess: Note that the ball's speed as it passes the window on its way down is the same as the speed with which it was tossed up, but in the opposite direction.

10.7. Model: Model the oxygen and the helium atoms as particles.
Visualize: We denote the oxygen and helium atoms by O and He, respectively. Note that the oxygen atom is four times heavier than the helium atom, so $m_\text{O} = 4\, m_\text{He}$.
Solve: The energy conservation equation $K_\text{O} = K_\text{He}$ is

$$\frac{1}{2}m_\text{O}v_\text{O}^2 = \frac{1}{2}m_\text{He}v_\text{He}^2 \Rightarrow (4\, m_\text{He})v_\text{O}^2 = m_\text{He}v_\text{He}^2 \Rightarrow \frac{v_\text{He}}{v_\text{O}} = 2.0$$

Assess: The result $v_\text{He} = 2v_\text{O}$, combined with the fact that $m_\text{He} = \frac{1}{4}m_\text{O}$, is a consequence of the way kinetic energy is defined: It is directly proportional to the mass and to the square of the speed.

10.9. Model: Model the skateboarder as a particle. Assuming that the track offers no rolling friction, the sum of the skateboarder's kinetic and gravitational potential energy does not change during his rolling motion.
Visualize:



The vertical displacement of the skateboarder is equal to the radius of the track.
Solve: The quantity $K + U_\text{g}$ is the same at the upper edge of the quarter-pipe track as it was at the bottom. The energy conservation equation $K_\text{f} + U_\text{gf} = K_\text{i} + U_\text{gi}$ is

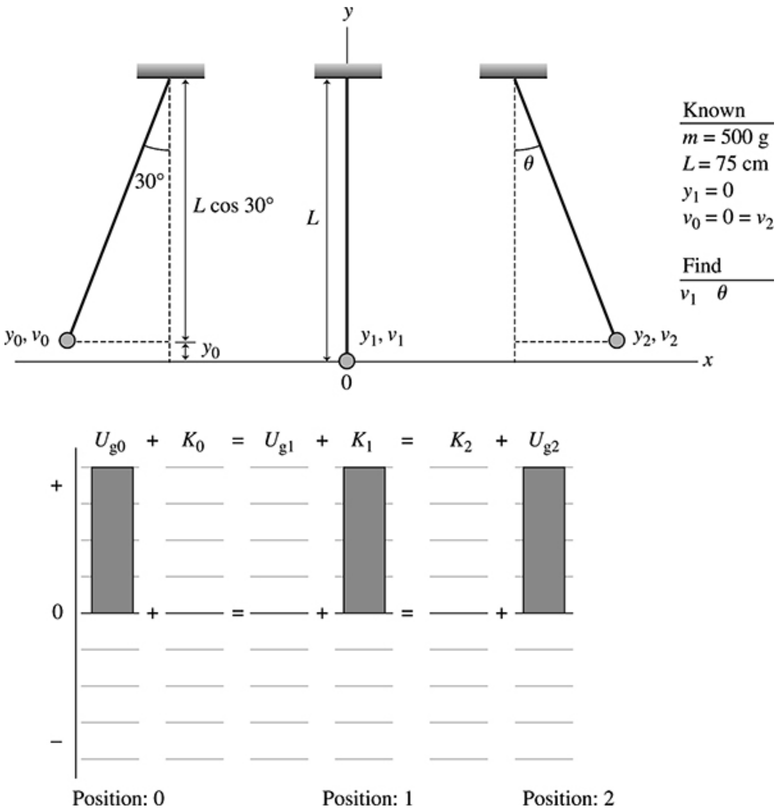
$$\frac{1}{2}mv_\text{f}^2 + mgy_\text{f} = \frac{1}{2}mv_\text{i}^2 + mgy_\text{i} \Rightarrow v_\text{i}^2 = v_\text{f}^2 + 2g(y_\text{f} - y_\text{i})$$

$$v_\text{i}^2 = (0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(3.0\text{ m} - 0\text{ m}) = 58.8\text{ m}^2/\text{s}^2 \Rightarrow v_\text{i} = 7.7\text{ m/s}$$

Assess: Note that we did not need to know the skateboarder's mass, as is the case with free-fall motion.

10.11. Model: In the absence of frictional and air-drag effects, the sum of the kinetic and gravitational potential energy does not change as the pendulum swings from one side to the other.

Visualize:



The figure shows the pendulum's before-and-after pictorial representation for the two situations described in parts (a) and (b).

Solve: (a) The quantity $K+U_g$ is the same at the lowest point of the trajectory as it was at the highest point. Thus, $K_1+U_{g1}=K_0+U_{g0}$ means

$$\frac{1}{2}mv_1^2+mgy_1=\frac{1}{2}mv_0^2+mgy_0 \Rightarrow v_1^2+2gy_1=v_0^2+2gy_0$$

$$\Rightarrow v_1^2+2g(0\text{ m})=(0\text{ m/s})^2+2gy_0 \Rightarrow v_1=\sqrt{2gy_0}$$

From the pictorial representation, we find that $y_0=L-L\cos 30°$. Thus,

$$v_1=\sqrt{2gL(1-\cos 30°)}=\sqrt{2(9.8\text{ m/s}^2)(0.75\text{ m})(1-\cos 30°)}=1.403\text{ m/s}$$

The speed at the lowest point is 1.40 m/s.

(b) Since the quantity $K+U_g$ does not change, $K_2+U_{g2}=K_1+U_{g1}$. We have

$$\frac{1}{2}mv_2^2+mgy_2=\frac{1}{2}mv_1^2+mgy_1 \Rightarrow y_2=\left(v_1^2-v_2^2\right)/2g$$

$$\Rightarrow y_2=[(1.403\text{ m/s})^2-(0\text{ m/s})^2]/(2\times 9.8\text{ m/s}^2)=0.100\text{ m}$$

Since $y_2=L-L\cos\theta$, we obtain

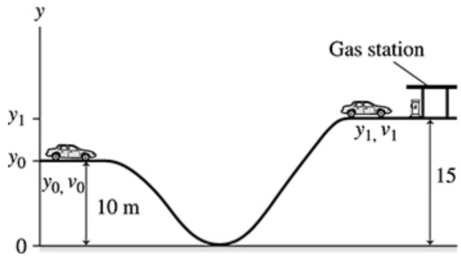
$$\cos\theta=\frac{L-y_2}{L}=\frac{(0.75\text{ m})-(0.10\text{ m})}{(0.75\text{ m})}=0.8667 \Rightarrow \theta=\cos^{-1}(0.8667)=30°$$

That is, the pendulum swings to the other side by $30°$.

Assess: The swing angle is the same on either side of the rest position. This result is a consequence of the fact that the sum of the kinetic and gravitational potential energy does not change. This is shown as well in the energy bar chart in the figure.

10.13. Model: Model the car as a particle with zero rolling friction. The sum of the kinetic and gravitational potential energy, therefore, does not change during the car's motion.

Visualize:



Solve: The initial energy of the car is

$$K_0 + U_{g0} = \frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}(1500 \text{ kg})(10.0 \text{ m/s})^2 + (1500 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m}) = 2.22\times10^5 \text{ J}$$

The car increases its height to 15 m at the gas station. The conservation of energy equation $K_0 + U_{g0} = K_1 + U_{g1}$ is

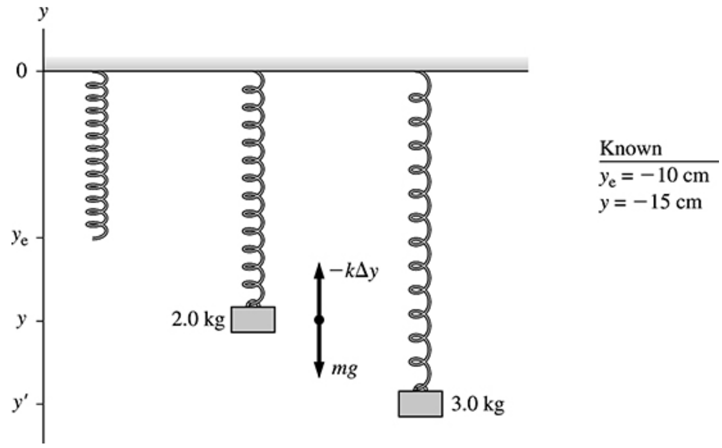
$$2.22\times10^5 \text{ J} = \frac{1}{2}mv_1^2 + mgy_1 \Rightarrow 2.22\times10^5 \text{ J} = \frac{1}{2}(1500 \text{ kg})v_1^2 + (1500 \text{ kg})(9.8 \text{ m/s}^2)(15 \text{ m})$$

$$\Rightarrow v_1 = 1.41 \text{ m/s}$$

Assess: A lower speed at the gas station is reasonable because the car has decreased its kinetic energy and increased its potential energy compared to its starting values.

10.16. Model: Assume an ideal spring that obeys Hooke's law.

Visualize:



Solve: **(a)** The spring force on the 2.0 kg mass is $F_{sp} = -k\Delta y$. Notice that Δy is negative, so F_{sp} is positive. This force is equal to *mg*, because the 2.0 kg mass is at rest. We have $-k\Delta y = mg$. Solving for *k*:

$$k = -(mg/\Delta y) = -(2.0 \text{ kg})(9.8 \text{ m/s}^2)/(-0.15 \text{ m} - (-0.10 \text{ m})) = 392 \text{ N/m}$$

The spring constant is 3.9×10^2 N/m.

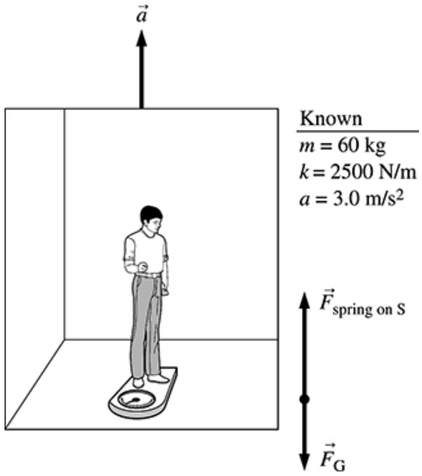
(b) Again using $-k\Delta y = mg$:

$$\Delta y = -mg/k = -(3.0 \text{ kg})(9.8 \text{ m/s}^2)/(392 \text{ N/m})$$

$$y' - y_e = -0.075 \text{ m} \Rightarrow y' = y_e - 0.075 \text{ m} = -0.10 \text{ m} - 0.075 \text{ m} = -0.175 \text{ m} = -17.5 \text{ cm}$$

The length of the spring is 17.5 cm when a mass of 3.0 kg is attached to the spring. The *position* of the end of the spring is negative because it is below the origin, but length must be a positive number.

10.18. Model: Model the student (S) as a particle and the spring as obeying Hooke's law.
Visualize:



Solve: According to Newton's second law the force on the student is

$$\sum (F_{\text{on S}})_y = F_{\text{spring on S}} - F_G = ma_y$$

$$\Rightarrow F_{\text{spring on S}} = F_G + ma_y = mg + ma_y = (60 \text{ kg})(9.8 \text{ m/s}^2 + 3.0 \text{ m/s}^2) = 768 \text{ N}$$

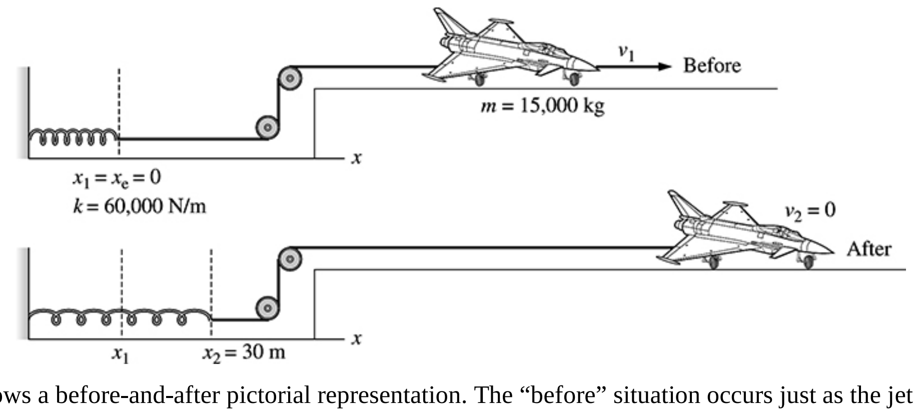
Since $F_{\text{spring on S}} = F_{\text{S on spring}} = k\Delta y$, $k\Delta y = 768$ N. This means $\Delta y = (768 \text{ N})/(2500 \text{ N/m}) = 0.31$ m.

10.19. Model: Assume an ideal spring that obeys Hooke's law.
Solve: The elastic potential energy of a spring is defined as $U_s = \frac{1}{2}k(\Delta s)^2$, where Δs is the magnitude of the stretching or compression relative to the unstretched or uncompressed length. We have $\Delta s = 20$ cm = 0.20 m and $k = 500$ N/m. This means

$$U_s = \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}(500 \text{ N/m})(0.20 \text{ m}) = 10 \text{ J}$$

Assess: Since Δs is squared, U_s is positive for a spring that is either compressed or stretched. U_s is zero when the spring is in its equilibrium position.

10.24. Model: Model the jet plane as a particle, and the spring as an ideal that obeys Hooke's law. We will also assume zero rolling friction during the stretching of the spring, so that mechanical energy is conserved.
Visualize:



The figure shows a before-and-after pictorial representation. The "before" situation occurs just as the jet plane lands on the aircraft carrier and the spring is in its equilibrium position. We put the origin of our coordinate system at the right free end of the spring. This gives $x_1 = x_e = 0$ m. Since the spring stretches 30 m to stop the plane, $x_2 - x_e = 30$ m.
Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ for the spring-jet plane system is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

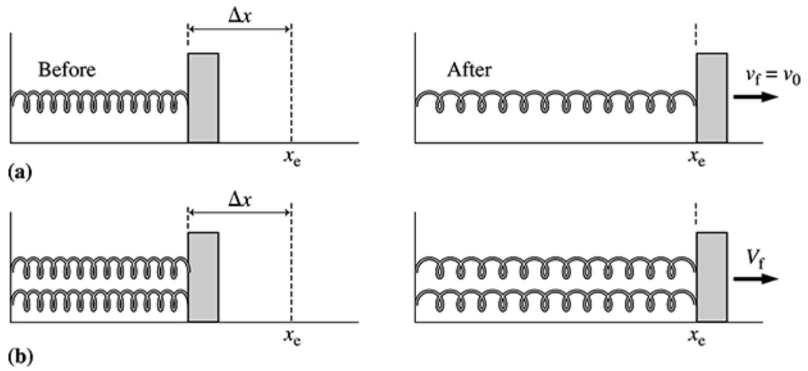
Using $v_2 = 0$ m/s, $x_1 = x_e = 0$ m, and $x_2 - x_e = 30$ m yields

$$\frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{m}}(x_2 - x_1) = \sqrt{\frac{60{,}000\ \text{N/m}}{15{,}000\ \text{kg}}}(30\ \text{m}) = 60\ \text{m/s}$$

Assess: A landing speed of 60 m/s or ≈120 mph is reasonable.

10.38. Model: Model the block as a particle and the springs as ideal springs obeying Hooke's law. There is no friction, hence the mechanical energy $K + U_s$ is conserved.

Visualize:



Note that $x_f = x_e$ and $x_i - x_e = \Delta x$. The before-and-after pictorial representations show that we put the origin of the coordinate system at the equilibrium position of the free end of the springs.

Solve: The conservation of energy equation $K_f + U_{sf} = K_i + U_{si}$ for the single spring is

$$\frac{1}{2}mv_f^2 + \frac{1}{2}k(x_f - x_e)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(x_i - x_e)^2$$

Using the value for v_f given in the problem, we get

$$\frac{1}{2}mv_0^2 + 0\ \text{J} = 0\ \text{J} + \frac{1}{2}k(\Delta x)^2 \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}k(\Delta x)^2$$

Conservation of energy for the two-spring case:

$$\frac{1}{2}mV_f^2 + 0\ \text{J} = 0\ \text{J} + \frac{1}{2}k(x_i - x_e)^2 + \frac{1}{2}k(x_i - x_e)^2 \qquad \frac{1}{2}mV_f^2 = k(\Delta x)^2$$

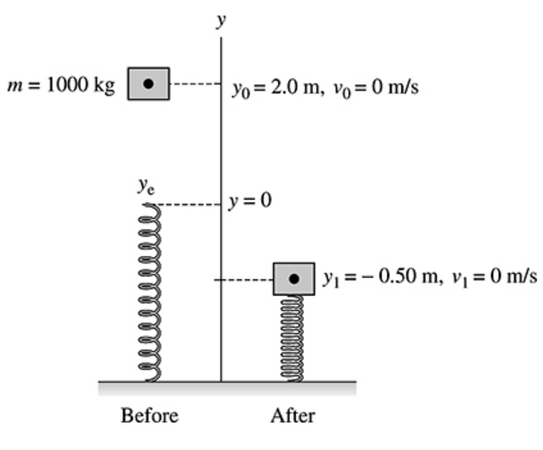
Using the result of the single-spring case,

$$\frac{1}{2}mV_f^2 = mv_0^2 \Rightarrow V_f = \sqrt{2}v_0$$

Assess: The block separates from the spring at the equilibrium position of the spring.

10.44. Model: Assume an ideal spring that obeys Hooke's law. Since this is a free-fall problem, the mechanical energy $K + U_g + U_s$ is conserved. Also, model the safe as a particle.

Visualize:



We have chosen to place the origin of our coordinate system at the free end of the spring, which is neither stretched nor compressed. The safe gains kinetic energy as it falls. The energy is then converted into elastic potential energy as the safe compresses the spring. The only two forces are gravity and the spring force, which are both conservative, so energy is conserved throughout the process. This means that the initial energy—as the safe is released—equals the final energy—when the safe is at rest and the spring is fully compressed.

Solve: The conservation of energy equation $K_1 + U_{g1} + U_{s1} = K_0 + U_{g0} + U_{s0}$ is

$$\frac{1}{2}mv_1^2 + mg(y_1 - y_e) + \frac{1}{2}k(y_1 - y_e)^2 = \frac{1}{2}mv_0^2 + mg(y_0 - y_e) + \frac{1}{2}k(y_e - y_e)^2$$

Using $v_1 = v_0 = 0$ m/s and $y_e = 0$ m, the above equation simplifies to

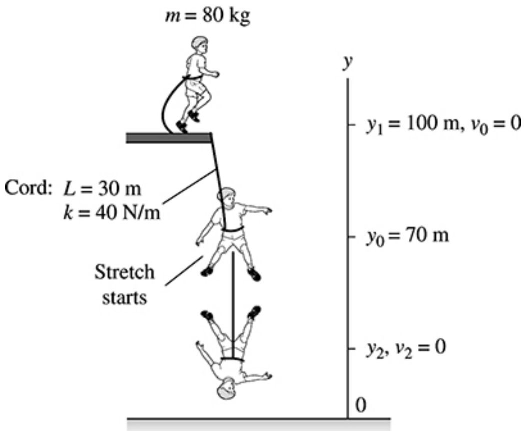
$$mgy_1 + \frac{1}{2}ky_1^2 = mgy_0$$

$$\Rightarrow k = \frac{2mg(y_0 - y_1)}{y_1^2} = \frac{2(1000\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m} - (-0.50\text{ m}))}{(-0.50\text{ m})^2} = 1.96\times10^5\text{ N/m}$$

Assess: By equating energy at these two points, we do not need to find how fast the safe was moving when it hit the spring.

10.70. Model: Choose yourself + spring + earth as the system. There are no forces from outside this system, so it is an isolated system. The interaction forces within the system are the spring force of the bungee cord and the gravitational force. These are both conservative forces, so mechanical energy is conserved.

Visualize:



We can equate the system's initial energy, as you step off the bridge, to its final energy when you reach the lowest point. We do *not* need to compute your speed at the point where the cord starts to stretch. We do, however, need to note that the end of the *unstretched* cord is at $y_0 = y_1 - 30\text{ m} = 70\text{ m}$, so $U_{2s} = \frac{1}{2}k(y_2 - y_0)^2$. Also note that $U_{1s} = 0$, since the cord is not stretched. The energy conservation equation is

$$K_2 + U_{2g} + U_{2s} = K_1 + U_{1g} + U_{1s} \Rightarrow 0\text{ J} + mgy_2 + \frac{1}{2}k(y_2 - y_0)^2 = 0\text{ J} + mgy_1 + 0\text{ J}$$

Multiply out the square of the binomial and rearrange:

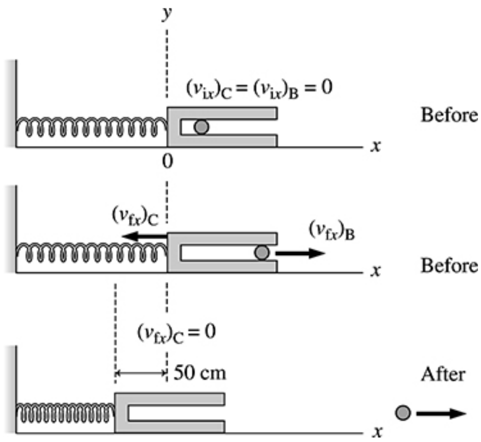
$$mgy_2 + \frac{1}{2}ky_2^2 - ky_0y_2 + \frac{1}{2}ky_0^2 = mgy_1$$

$$\Rightarrow y_2^2 + \left(\frac{2mg}{k} - 2y_0\right)y_2 + \left(y_0^2 - \frac{2mgy_1}{k}\right) = y_2^2 - 100.8y_2 + 980 = 0$$

This is a quadratic equation with roots 89.9 m and 10.9 m. The first is not physically meaningful because it is a height above the point where the cord started to stretch. So we find that your distance from the water when the bungee cord stops stretching is 10.9 m.

10.72. Model: Assume an ideal, massless spring that obeys Hooke's law. Let us also assume that the cannon (C) fires balls (B) horizontally and that the spring is directly behind the cannon to absorb all motion.

Visualize:



The before-and-after pictorial representation is shown, with the origin of the coordinate system located at the spring's free end when the spring is neither compressed nor stretched. This free end of the spring is just behind the cannon.

Solve: The momentum conservation equation $p_{fx} = p_{ix}$ is

$$m_B(v_{fx})_B + m_C(v_{fx})_C = m_B(v_{ix})_B + m_C(v_{ix})_C$$

Since the initial momentum is zero,

$$(v_{fx})_B = -\frac{m_C}{m_B}(v_{fx})_C = -\left(\frac{200\text{ kg}}{10\text{ kg}}\right)(v_{fx})_C = -20(v_{fx})_C$$

The mechanical energy conservation equation for the cannon + spring $K_f + U_{sf} = K_i + U_{si}$ is

$$\frac{1}{2}m(v_f)_C^2 + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m(v_i)_C^2 + 0\text{ J} \Rightarrow 0\text{ J} + \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}m(v_{fx})_C^2$$

$$\Rightarrow (v_{fx})_C = \pm\sqrt{\frac{k}{m}}\Delta x = \pm\sqrt{\frac{(20{,}000\text{ N/m})}{200\text{ kg}}}(0.50\text{ m}) = \pm 5.0\text{ m/s}$$

To make this velocity physically correct, we retain the minus sign with $(v_{fx})_C$. Substituting into the momentum conservation equation yields:

$$(v_{fx})_B = -20(-5.0\text{ m/s}) = 100\text{ m/s}$$