Chapter 14, Conceptual Questions

14.1. The period of a block oscillating on a spring is $T = 2\pi\sqrt{m/k}$. We are told that $T_1 = 2.0$ s.

(a) In this case the mass is doubled: $m_2 = 2m_1$.

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{m_2/k}}{2\pi\sqrt{m_1/k}} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{2m_1}{m_1}} = \sqrt{2}$$

So $T_2 = \sqrt{2}T_1 = \sqrt{2}(2.0\text{ s}) = 2.8$ s.

(b) In this case the spring constant is doubled: $k_2 = 2k_1$.

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{m/k_2}}{2\pi\sqrt{m/k_1}} = \sqrt{\frac{k_1}{k_2}} = \sqrt{\frac{k_1}{2k_1}} = \frac{1}{\sqrt{2}}$$

So $T_2 = T_1/\sqrt{2} = (2.0\text{ s})/\sqrt{2} = 1.4$ s.

(c) The formula for the period does not contain the amplitude; that is, the period is independent of the amplitude. Changing (in particular, doubling) the amplitude does not affect the period, so the new period is still 2.0 s.

It is equally important to understand what *doesn't* appear in a formula. It is quite startling, really, the first time you realize it, that the amplitude doesn't affect the period. But this is crucial to the idea of simple harmonic motion. Of course, if the spring is stretched too far, out of its linear region, then the amplitude would matter.

14.2. The period of a simple pendulum is $T = 2\pi\sqrt{L/g}$. We are told that $T_1 = 2.0$ s.

(a) In this case the mass is doubled: $m_2 = 2m_1$. However, the mass does not appear in the formula for the period of a pendulum; that is, the period does not depend on the mass. Therefore the period is still 2.0 s.

(b) In this case the length is doubled: $L_2 = 2L_1$.

$$\frac{T_2}{T_1} = \frac{2\pi\sqrt{L_2/g}}{2\pi\sqrt{L_1/g}} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{2L_1}{L_1}} = \sqrt{2}$$

So $T_2 = \sqrt{2}T_1 = \sqrt{2}(2.0\text{ s}) = 2.8$ s.

(c) The formula for the period of a simple small-angle pendulum does not contain the amplitude; that is, the period is independent of the amplitude. Changing (in particular, doubling) the amplitude, as long as it is still small, does not affect the period, so the new period is still 2.0 s.

It is equally important to understand what *doesn't* appear in a formula. It is quite startling, really, the first time you realize it, that the amplitude (θ_{max}) doesn't affect the period. But this is crucial to the idea of simple harmonic motion. Of course, if the pendulum is swung too far, out of its linear region, then the amplitude would matter. The amplitude *does* appear in the formula for a pendulum not restricted to small angles because the small-angle approximation is not valid; but then the motion is not simple harmonic motion.

14.3. **(a)** The amplitude is the maximum displacement obtained. From the graph, $A = 10$ cm.

(b) From the graph, the period $T = 2.0$ s. The angular frequency is thus $\omega = \dfrac{2\pi}{T} = 3.1$ rad/s.

(c) The phase constant specifies at what point the cosine function starts. From the graph, the starting point looks like $\frac{1}{2}A$. So at $t = 0$, $A\cos\phi = \frac{1}{2}A$ means $\phi_0 = \pm\frac{\pi}{3}(\pm 60°)$. Since the oscillator is moving to the left at $t = 0$, it is in the upper half of the circular-motion diagram, and we choose $\phi_0 = +\frac{\pi}{3}$.

14.4. **(a)** A position *vs* time graph plots $x(t) = A\cos(wt + \phi_0)$. The graph of $x(t)$ starts at $\frac{1}{2}A$ and is increasing. So at $t = 0$, $\frac{1}{2}A = A\cos\phi_0 \Rightarrow \phi_0 = \pm\frac{\pi}{3}$. We choose $\phi_0 = -\frac{\pi}{3}$ since the particle is moving to the right, indicating that it is in the bottom half of the circular-motion diagram.

(b) The phase at each point can be determined in the same manner as for part (a). For points 1 and 3, the amplitude is again $\frac{1}{2}A$. At point 1, the particle is moving to the right so $\phi_1 = -\frac{\pi}{3}$. At point 3, the particle is moving left, so $\phi_3 = +\frac{\pi}{3}$. At point 2, the amplitude is *A*, so $\cos\phi_2 = 1 \Rightarrow \phi_2 = 0$.

14.5. **(a)** A velocity-vs-time graph is a plot of $v(t) = -A\omega\sin(\omega t + \phi_0)$. At $t = 0$, the amplitude is $\frac{1}{2}v_{\max} = \frac{1}{2}A\omega = -A\omega\sin\phi_0$, so $\phi_0 = \sin^{-1}\left(-\frac{1}{2}\right) = \frac{7\pi}{6}$ or $-\frac{\pi}{6}$. Since $v(t = 0\text{ s})$ is increasing, we need $-\sin(\omega t + \phi_0)$ increasing at $t = 0$, so we choose $\phi_0 = \frac{7\pi}{6}$.

(b) The phase is $\phi = wt + \phi_0$. At points 1 and 3, $v(t) = -\frac{1}{2}v_{\max}$, so $\phi = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. At point 1, we need $-\sin\phi_1$, increasing, so $\phi_1 = \frac{5\pi}{6}$. At point 3, $-\sin\phi_3$ is decreasing, so $\phi_3 = \frac{\pi}{6}$. At point 2, the amplitude is $v_{\max}$, so $-\sin\phi_2 = 1$, thus $\phi_2 = -\frac{\pi}{2}$.

14.6. Energy is transferred back and forth between all potential energy at the extremes $\left(\frac{1}{2}kA^2\right)$ and all kinetic energy at the equilibrium point(s) $\left(\frac{1}{2}m{v_{\max}}^2\right)$. The equation does *not* say that the particle ever has amplitude *A* and speed $v_{\max}$ at the same time. The equation relates expressions for the energy at two different times.

14.8. One expression for the total energy is $E = \frac{1}{2}kA^2$, which occurs at the turning points when $v(t) = 0$. If the total energy is doubled,

$$2E = \frac{1}{2}k(A')^2 = \frac{1}{2}k\left(\sqrt{2}A\right)^2$$

Thus the new amplitude is $A' = \sqrt{2}A = \sqrt{2}(20\text{ cm}) = 28\text{ cm}$.

14.9. One expression for the total energy is $E = \frac{1}{2}mv_{\max}^2$, which occurs at the equilibrium point when $U = 0$. If the total energy is doubled,

$$2E = \frac{1}{2}m(v'_{\max})^2 = \frac{1}{2}m\left(\sqrt{2}v_{\max}\right)^2$$

Thus the new maximum speed is $v'_{\max} = \sqrt{2}v_{\max} = \sqrt{2}(20\text{ cm/s}) = 28\text{ cm/s}$

14.10. The time constant $\tau = \frac{m}{b}$ decreases as *b* increases, meaning energy is removed from the oscillator more quickly since $E(t) = E_0 e^{-t/\tau}$.

(a) The medium is more resistive.
(b) The oscillations damp out more quickly.
(c) The time constant is decreased.

14.11. **(a)** *T*, the period, is the time for each cycle of the motion, the time required for the motion to repeat itself. τ, the damping time constant, is the time required for the amplitude of a damped oscillator to decrease to $e^{-1} \approx 37\%$ of its original value.
(b) τ, the damping time constant, is the time required for the amplitude of a damped oscillator to decrease to $e^{-1} \approx 37\%$ of its original value, while $t_{1/2}$, the half-life, is the time required for the amplitude of a damped oscillator to decrease to 50% of its original value.

14.12. Natural frequency is the frequency that an oscillator will oscillate at on its own. You may drive an oscillator at a frequency other than its natural frequency. For example, if you are pushing a child in a swing, you build up the amplitude of the oscillation by driving the oscillator at its natural frequency. You can achieve resonance by driving an oscillator at its natural frequency.

Chapter 14, Exercises and Problems

14.1. **Solve:** The frequency generated by a guitar string is 440 Hz. The period is the inverse of the frequency, hence

$$T = \frac{1}{f} = \frac{1}{440 \text{ Hz}} = 2.27 \times 10^{-3} \text{ s} = 2.27 \text{ ms}$$

14.2. **Model:** The air-track glider oscillating on a spring is in simple harmonic motion.
Solve: The glider completes 10 oscillations in 33 s, and it oscillates between the 10 cm mark and the 60 cm mark.

(a) $$T = \frac{33 \text{ s}}{10 \text{ oscillations}} = 3.3 \text{ s/oscillation} = 3.3 \text{ s}$$

(b) $$f = \frac{1}{T} = \frac{1}{3.3 \text{ s}} = 0.303 \text{ Hz} \approx 0.30 \text{ Hz}$$

(c) $$\omega = 2\pi f = 2\pi(0.303 \text{ Hz}) = 1.90 \text{ rad/s}$$

(d) The oscillation from one side to the other is equal to $60 \text{ cm} - 10 \text{ cm} = 50 \text{ cm} = 0.50 \text{ m}$. Thus, the amplitude is $A = \frac{1}{2}(0.50 \text{ m}) = 0.25 \text{ m}$.

(e) The maximum speed is

$$v_{\max} = \omega A = \left(\frac{2\pi}{T}\right)A = (1.90 \text{ rad/s})(0.25 \text{ m}) = 0.48 \text{ m/s}$$

14.3. **Model:** The air-track glider attached to a spring is in simple harmonic motion.
Visualize: The position of the glider can be represented as $x(t) = A\cos\omega t$.
Solve: The glider is pulled to the right and released from rest at $t = 0$ s. It then oscillates with a period $T = 2.0$ s and a maximum speed $v_{\max} = 40 \text{ cm/s} = 0.40 \text{ m/s}$.

(a) $v_{\max} = \omega A$ and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0 \text{ s}} = \pi \text{ rad/s} \Rightarrow A = \frac{v_{\max}}{\omega} = \frac{0.40 \text{ m/s}}{\pi \text{ rad/s}} = 0.127 \text{ m} = 12.7 \text{ cm}$

(b) The glider's position at $t = 0.25$ s is

$$x_{0.25 \text{ s}} = (0.127 \text{ m})\cos\left[(\pi \text{ rad/s})(0.25 \text{ s})\right] = 0.090 \text{ m} = 9.0 \text{ cm}$$

14.5. **Model:** The oscillation is the result of simple harmonic motion.
Visualize: Please refer to Figure EX14.5.
Solve: **(a)** The amplitude $A = 20$ cm.
(b) The period $T = 4.0$ s, thus

$$f = \frac{1}{T} = \frac{1}{4.0 \text{ s}} = 0.25 \text{ Hz}$$

(c) The position of an object undergoing simple harmonic motion is $x(t) = A\cos(\omega t + \phi_0)$. At $t = 0$ s, $x_0 = 10$ cm. Thus,

$$10 \text{ cm} = (20 \text{ cm})\cos\phi_0 \Rightarrow \phi_0 = \cos^{-1}\left(\frac{10 \text{ cm}}{20 \text{ cm}}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \pm\frac{\pi}{3} \text{ rad} = \pm 60°$$

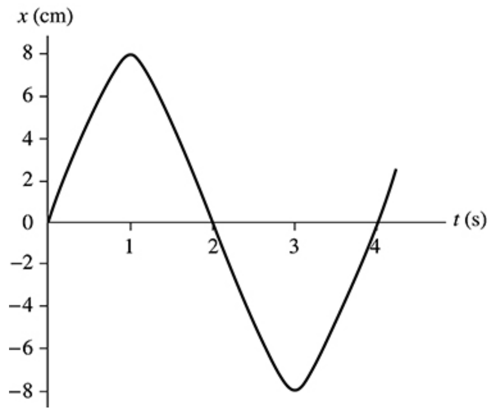
Because the object is moving to the right at $t = 0$ s, it is in the lower half of the circular motion diagram and thus must have a phase constant between π and 2π radians. Therefore, $\phi_0 = -\frac{\pi}{3} \text{ rad} = -60°$.

14.7. **Visualize:** A phase constant of $-\frac{\pi}{2}$ implies that the object that undergoes simple harmonic motion is in the lower half of the circular motion diagram. That is, the object is moving to the right.

Solve: The position of the object is given by the equation

$$x(t) = A\cos(\omega t + \phi_0) = A\cos(2\pi ft + \phi_0) = (8.0\text{ cm})\cos\left[\left(\frac{\pi}{2}\text{ rad/s}\right)t - \frac{\pi}{2}\text{ rad}\right]$$

The amplitude is $A = 8.0$ cm and the period is $T = 1/f = 4.0$ s. With $\phi_0 = -\pi/2$ rad, x starts at 0 cm and is moving to the right (getting more positive).



Assess: As we see from the graph, the object starts out moving to the right.

14.9. Solve: The position of the object is given by the equation

$$x(t) = A\cos(\omega t + \phi_0)$$

The amplitude $A = 8.0$ cm. The angular frequency $\omega = 2\pi f = 2\pi(0.50\text{ Hz}) = \pi$ rad/s. Since at $t = 0$ it has its most negative velocity, it must be at the equilibrium point $x = 0$ cm and moving to the left, so $\phi_0 = \frac{\pi}{2}$. Thus

$$x(t) = (8.0\text{ cm})\cos[(\pi\text{ rad/s})t + \frac{\pi}{2}\text{ rad}]$$

14.11. Model: The block attached to the spring is in simple harmonic motion.
Solve: The period of an object attached to a spring is

$$T = 2\pi\sqrt{\frac{m}{k}} = T_0 = 2.0\text{ s}$$

where m is the mass and k is the spring constant.
(a) For mass $= 2m$,

$$T = 2\pi\sqrt{\frac{2m}{k}} = (\sqrt{2})T_0 = 2.8\text{ s}$$

(b) For mass $\frac{1}{2}m$,

$$T = 2\pi\sqrt{\frac{\frac{1}{2}m}{k}} = T_0/\sqrt{2} = 1.41\text{ s}$$

(c) The period is independent of amplitude. Thus $T = T_0 = 2.0$ s
(d) For a spring constant $= 2k$,

$$T = 2\pi\sqrt{\frac{m}{2k}} = T_0/\sqrt{2} = 1.41\text{ s}$$

14.13. Model: The mass attached to the spring oscillates in simple harmonic motion.
Solve: **(a)** The period $T = 1/f = 1/2.0\text{ Hz} = 0.50$ s.
(b) The angular frequency $\omega = 2\pi f = 2\pi(2.0\text{ Hz}) = 4\pi$ rad/s.
(c) Using energy conservation

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}kx_0^2 + \tfrac{1}{2}mv_{0x}^2$$

Using $x_0 = 5.0$ cm, $v_{0x} = -30$ cm/s and $k = m\omega^2 = (0.200 \text{ kg})(4\pi \text{ rad/s})^2$, we get $A = 5.54$ cm.

(d) To calculate the phase constant ϕ_0,

$$A\cos\phi_0 = x_0 = 5.0 \text{ cm}$$
$$\Rightarrow \phi_0 = \cos^{-1}\left(\frac{5.0 \text{ cm}}{5.54 \text{ cm}}\right) = 0.45 \text{ rad}$$

(e) The maximum speed is $v_{max} = \omega A = (4\pi \text{ rad/s})(5.54 \text{ cm}) = 70 \text{ cm/s}$.

(f) The maximum acceleration is

$$a_{max} = \omega^2 A = \omega(\omega A) = (4\pi \text{ rad/s})(70 \text{ cm/s}) = 8.8 \text{ m/s}^2$$

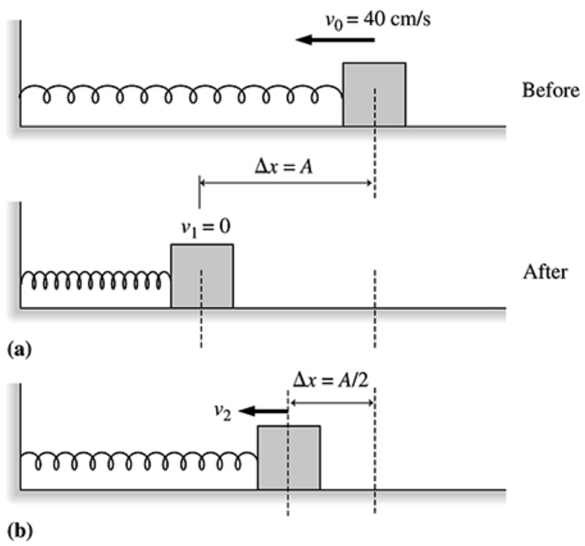
(g) The total energy is $E = \tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}(0.200 \text{ kg})(0.70 \text{ m/s})^2 = 0.049$ J.

(h) The position at $t = 0.40$ s is

$$x_{0.4 \text{ s}} = (5.54 \text{ cm})\cos\left[(4\pi \text{ rad/s})(0.40 \text{ s}) + 0.45 \text{ rad}\right] = +3.8 \text{ cm}$$

14.15. Model: The block attached to the spring is in simple harmonic motion.

Visualize:



Solve: (a) The conservation of mechanical energy equation $K_f + U_{sf} = K_i + U_{si}$ is

$$\tfrac{1}{2}mv_1^2 + \tfrac{1}{2}k(\Delta x)^2 = \tfrac{1}{2}mv_0^2 + 0 \text{ J} \Rightarrow 0 \text{ J} + \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_0^2 + 0 \text{ J}$$
$$\Rightarrow A = \sqrt{\frac{m}{k}}v_0 = \sqrt{\frac{1.0 \text{ kg}}{16 \text{ N/m}}}(0.40 \text{ m/s}) = 0.10 \text{ m} = 10.0 \text{ cm}$$

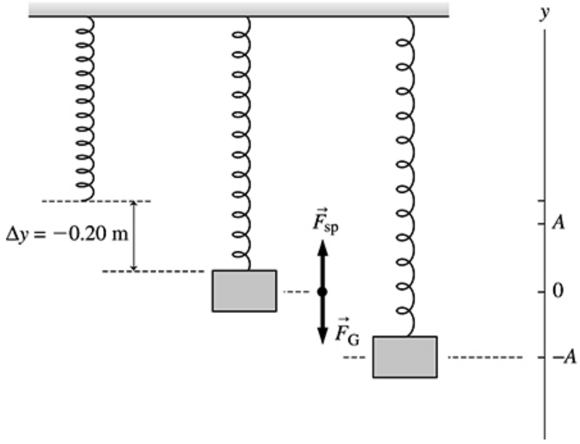
(b) We have to find the velocity at a point where $x = A/2$. The conservation of mechanical energy equation $K_2 + U_{s2} = K_i + U_{si}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}mv_0^2 + 0 \text{ J} \Rightarrow \frac{1}{2}mv_2^2 = \frac{1}{2}mv_0^2 - \frac{1}{4}\left(\frac{1}{2}kA^2\right) = \frac{1}{2}mv_0^2 - \frac{1}{4}\left(\frac{1}{2}mv_0^2\right) = \frac{3}{4}\left(\frac{1}{2}mv_0^2\right)$$
$$\Rightarrow v_2 = \sqrt{\frac{3}{4}}v_0 = \sqrt{\frac{3}{4}}(0.40 \text{ m/s}) = 0.346 \text{ m/s}$$

The velocity is 35 cm/s.

14.16. Model: The vertical oscillations constitute simple harmonic motion.

Visualize:



Solve: **(a)** At equilibrium, Newton's first law applied to the physics book is

$$(F_{sp})_y - mg = 0 \text{ N} \Rightarrow -k\Delta y - mg = 0 \text{ N}$$
$$\Rightarrow k = -mg/\Delta y = -(0.500 \text{ kg})(9.8 \text{ m/s}^2)/(-0.20 \text{ m}) = 24.5 \text{ N/m}$$

(b) To calculate the period:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{24.5 \text{ N/m}}{0.500 \text{ kg}}} = 7.0 \text{ rad/s and } T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{7.0 \text{ rad/s}} = 0.90 \text{ s}$$

(c) The maximum speed is

$$v_{max} = A\omega = (0.10 \text{ m})(7.0 \text{ rad/s}) = 0.70 \text{ m/s}$$

Maximum speed occurs as the book passes through the equilibrium position.

14.17. **Model:** The vertical oscillations constitute simple harmonic motion.

Solve: To find the oscillation frequency using $\omega = 2\pi f = \sqrt{k/m}$, we first need to find the spring constant k. In equilibrium, the weight mg of the block and the spring force $k\Delta L$ are equal and opposite. That is, $mg = k\Delta L \Rightarrow k = mg/\Delta L$. The frequency of oscillation f is thus given as

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{mg/\Delta L}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\Delta L}} = \frac{1}{2\pi}\sqrt{\frac{9.8 \text{ m/s}^2}{0.020 \text{ m}}} = 3.5 \text{ Hz}$$

14.19. **Model:** Assume a small angle of oscillation so there is simple harmonic motion.
Solve: The period of the pendulum is

$$T_0 = 2\pi\sqrt{\frac{L_0}{g}} = 4.0 \text{ s}$$

(a) The period is independent of the mass and depends only on the length. Thus $T = T_0 = 4.0$ s.

(b) For a new length $L = 2L_0$,

$$T = 2\pi\sqrt{\frac{2L_0}{g}} = \sqrt{2}T_0 = 5.7 \text{ s}$$

(c) For a new length $L = L_0/2$,

$$T = 2\pi\sqrt{\frac{L_0/2}{g}} = \frac{1}{\sqrt{2}}T_0 = 2.8 \text{ s}$$

(d) The period is independent of the amplitude as long as there is simple harmonic motion. Thus $T = 4.0$ s.

14.20. Model: The pendulum undergoes simple harmonic motion.
Solve: **(a)** The amplitude is 0.10 rad.
(b) The frequency of oscillations is

$$f = \frac{\omega}{2\pi} = \frac{5}{2\pi}\ \text{Hz} = 0.796\ \text{Hz}$$

(c) The phase constant $\phi = \pi$ rad.
(d) The length can be obtained from the period:

$$\omega = 2\pi f = \sqrt{\frac{g}{L}} \Rightarrow L = \left(\frac{1}{2\pi f}\right)^2 g = \left(\frac{1}{2\pi(0.796\ \text{Hz})}\right)^2 \left(9.8\ \text{m/s}^2\right) = 0.392\ \text{m}$$

(e) At $t = 0$ s, $\theta_0 = (0.10\ \text{rad})\cos(\pi) = -0.10$ rad. To find the initial condition for the angular velocity we take the derivative of the angular position:

$$\theta(t) = (0.10\ \text{rad})\cos(5t + \pi) \Rightarrow \frac{d\theta(t)}{dt} = -(0.10\ \text{rad})(5)\sin(5t + \pi)$$

At $t = 0$ s, $(d\theta/dt)_0 = (-0.50\ \text{rad})\sin(\pi) = 0$ rad/s.
(f) At $t = 2.0$ s, $\theta_{2.0} = (0.10\ \text{rad})\cos\left(5(2.0\ \text{s}) + \pi\right) = 0.084$ rad.

14.21. Model: Assume the small-angle approximation so there is simple harmonic motion.
Solve: The period is $T = 12\ \text{s}/10\ \text{oscillations} = 1.20\ \text{s}$ and is given by the formula

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow L = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{1.20\ \text{s}}{2\pi}\right)^2 \left(9.8\ \text{m/s}^2\right) = 36\ \text{cm}$$

14.23. Model: Assume the pendulum to have small-angle oscillations. In this case, the pendulum undergoes simple harmonic motion.
Solve: Using the formula $g = GM/R^2$, the periods of the pendulums on the moon and on the earth are

$$T_{\text{earth}} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L_{\text{earth}}R_{\text{earth}}^2}{GM_{\text{earth}}}} \text{ and } T_{\text{moon}} = 2\pi\sqrt{\frac{L_{\text{moon}}R_{\text{moon}}^2}{GM_{\text{moon}}}}$$

Because $T_{\text{earth}} = T_{\text{moon}}$,

$$2\pi\sqrt{\frac{L_{\text{earth}}R_{\text{earth}}^2}{GM_{\text{earth}}}} = 2\pi\sqrt{\frac{L_{\text{moon}}R_{\text{moon}}^2}{GM_{\text{moon}}}} \Rightarrow L_{\text{moon}} = \left(\frac{M_{\text{moon}}}{M_{\text{earth}}}\right)\left(\frac{R_{\text{earth}}}{R_{\text{moon}}}\right)^2 L_{\text{earth}}$$

$$= \left(\frac{7.36\times10^{22}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}\right)\left(\frac{6.37\times10^{6}\ \text{m}}{1.74\times10^{6}\ \text{m}}\right)^2 (2.0\ \text{m}) = 33\ \text{cm}$$

14.24. Model: Assume a small angle of oscillation so that the pendulum has simple harmonic motion.
Solve: The time periods of the pendulums on the earth and on Mars are

$$T_{\text{earth}} = 2\pi\sqrt{\frac{L}{g_{\text{earth}}}} \text{ and } T_{\text{Mars}} = 2\pi\sqrt{\frac{L}{g_{\text{Mars}}}}$$

Dividing these two equations,

$$\frac{T_{\text{earth}}}{T_{\text{Mars}}} = \sqrt{\frac{g_{\text{Mars}}}{g_{\text{earth}}}} \Rightarrow g_{\text{Mars}} = g_{\text{earth}}\left(\frac{T_{\text{earth}}}{T_{\text{Mars}}}\right)^2 = \left(9.8\ \text{m/s}^2\right)\left(\frac{1.50\ \text{s}}{2.45\ \text{s}}\right)^2 = 3.67\ \text{m/s}^2$$

14.25. Visualize: Please refer to Figure Ex14.25.
Solve: The mass of the wrench can be obtained from the length that it stretches the spring. From Equation 14.41,

$$\Delta L = \frac{mg}{k} \Rightarrow m = \frac{k\Delta L}{g} = \frac{360 \text{ N/m}(0.030 \text{ m})}{9.8 \text{ m/s}^2} = 1.10 \text{ kg}$$

When swinging on a hook the wrench is a physical pendulum. From Equation 14.52,

$$2\pi f = \sqrt{\frac{mgl}{I}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{mgl}{I}}$$

From the figure, $l = 0.14$ m. Thus

$$I = \left(\frac{T}{2\pi}\right)^2 mgl = \left(\frac{0.90 \text{ s}}{2\pi}\right)^2 (1.10 \text{ kg})(9.8 \text{ m/s}^2)(0.14 \text{ m}) = 3.1\times10^{-2} \text{ kg m}^2$$

14.26. Model: The spider is in simple harmonic motion.
Solve: Your tapping is a driving frequency. Largest amplitude at $f_{\text{ext}} = 1.0$ Hz means that this is the resonance frequency, so $f_0 = f_{\text{ext}} = 1.0$ Hz. That is, the spider's natural frequency of oscillation f_0 is 1.0 Hz and $\omega_0 = 2\pi f_0 = 2\pi$ rad/s. We have

$$\omega_0 = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega_0^2 = (0.0020 \text{ kg})(2\pi \text{ rad/s})^2 = 0.079 \text{ N/m}$$

14.29. Model: The pendulum is a damped oscillator.
Solve: The period of the pendulum and the number of oscillations in 4 hours are calculated as follows:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{15.0 \text{ m}}{9.8 \text{ m/s}^2}} = 7.773 \text{ s} \Rightarrow N_{\text{osc}} = \frac{4(3600 \text{ s})}{7.773 \text{ s}} = 1853$$

The amplitude of the pendulum as a function of time is $A(t) = Ae^{-bt/2m}$. The exponent of this expression can be calculated to be

$$-\frac{bt}{2m} = -\frac{(0.010 \text{ kg/s})(4\times3600 \text{ s})}{2(110 \text{ kg})} = -0.6545$$

We have $A(t) = (1.50 \text{ m})e^{-0.6545} = 0.780$ m.

14.43. Model: The transducer undergoes simple harmonic motion.
Solve: Newton's second law for the transducer is

$$F_{\text{restoring}} = ma_{\max} \Rightarrow 40{,}000 \text{ N} = (0.10\times10^{-3} \text{ kg})a_{\max} \Rightarrow a_{\max} = 4.0\times10^8 \text{ m/s}^2$$

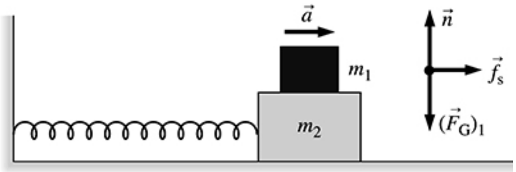
Because $a_{\max} = \omega^2 A$,

$$A = \frac{a_{\max}}{\omega^2} = \frac{4.0\times10^8 \text{ m/s}^2}{\left[2\pi(1.0\times10^6 \text{ Hz})\right]^2} = 1.01\times10^{-5} \text{ m} = 10.1 \,\mu\text{m}$$

(b) The maximum velocity is

$$v_{\max} = \omega A = 2\pi(1.0\times10^6 \text{ Hz})(1.01\times10^{-5} \text{ m}) = 64 \text{ m/s}$$

14.51. Model: The two blocks are in simple harmonic motion, without the upper block slipping. We will also apply the model of static friction between the two blocks.
Visualize:



Solve: The net force acting on the upper block m_1 is the force of friction due to the lower block m_2. The model of static friction gives the maximum force of static friction as

$$(f_s)_{max} = \mu_s n = \mu_s(m_1 g) = m_1 a_{max} \Rightarrow a_{max} = \mu_s g$$

Using $\mu_s = 0.5$, $a_{max} = \mu_s g = (0.5)(9.8\ \text{m/s}^2) = 4.9\ \text{m/s}^2$. That is, the two blocks will ride together if the maximum acceleration of the system is equal to or less than a_{max}. We can calculate the maximum value of *A* as follows:

$$a_{max} = \omega^2 A_{max} = \frac{k}{m_1 + m_2} A_{max} \Rightarrow A_{max} = \frac{a_{max}(m_1 + m_2)}{k} = \frac{(4.9\ \text{m/s}^2)(1.0\ \text{kg} + 5.0\ \text{kg})}{50\ \text{N/m}} = 0.59\ \text{m}$$

14.57. Model: The mass is a particle and the string is massless.

Solve: Equation 14.52 is

$$\omega = \sqrt{\frac{Mgl}{I}}$$

The moment of inertia of the mass on a string is $I = Ml^2$, where *l* is the length of the string. Thus

$$\omega = \sqrt{\frac{Mgl}{Ml^2}} = \sqrt{\frac{g}{l}}$$

This is Equation 14.49 with $L = l$.

Assess: Equation 14.49 is really a specific case of the more general physical pendulum described by Equation 14.52.

14.59. Model: The circular hoop can be modeled as a cylindrical hoop and its moment of inertia about the point of rotation found with the parallel-axis theorem.

Visualize: Please refer to Figure P14.59.

Solve: Using the parallel-axis theorem, the moment of inertia of the cylindrical hoop about the rotation point is

$$I = MR^2 + MR^2 = 2MR^2$$

The frequency of small oscillations is given by Equation 14.52.

$$f = \frac{1}{2\pi}\sqrt{\frac{Mgl}{I}}$$

The center of mass of the hoop is its center, so $l = R$. Thus

$$f = \frac{1}{2\pi}\sqrt{\frac{MgR}{2MR^2}} = \frac{1}{2\pi}\sqrt{\frac{g}{2R}}$$