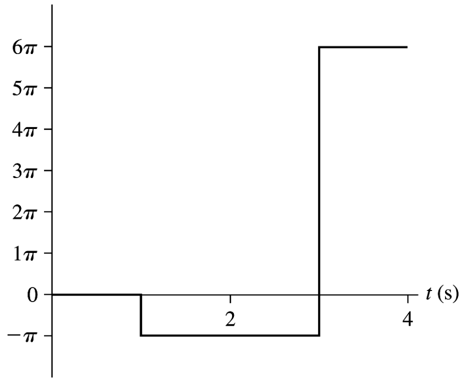
Chapter 4, Exercises and Problems [del 2]

4.20. Solve: (a) From $t=0$ s to $t=1$ s the particle does not rotate. From $t=1$ s to $t=3$ s, the particle rotates clockwise from the angular position 0 rad to -2π rad. Therefore, $\Delta\theta=-2\pi$ rad in two seconds, or $\omega=-\pi$ rad/s. From $t=3$ s to $t=4$ s the particle rotates counterclockwise from the angular position -2π rad to $+4\pi$ rad. Thus $\Delta\theta = 4\pi-(-2\pi)=6\pi$ rad and $\omega=+6\pi$ rad/s.

(b)



4.21. Solve: Since $\omega=(d\theta/dt)$ we have

$$\theta_f=\theta_i+\text{area under the } \omega\text{-versus-}t \text{ graph between } t_i \text{ and } t_f$$

From $t=0$ s to $t=2$ s, the area is $\frac{1}{2}(20 \text{ rad/s})(2 \text{ s})=20$ rad. From $t=2$ s to $t=4$ s, the area is $(20 \text{ rad/s})(2 \text{ s})=40$ rad. Thus, the area under the ω-versus-t graph during the total time interval of 4 s is 60 rad or $(60 \text{ rad})\times(1 \text{ rev}/2\pi\text{rad})=$
9.55 revolutions.

4.22. Solve: Since $\omega=(d\theta/dt)$ we have

$$\theta_f=\theta_i+\text{area under the } \omega \text{ versus } t \text{ graph between } t_i \text{ and } t_f$$

From $t=0$ s to s to $t=4$ s, the area is $\frac{1}{2}(20 \text{ rad/s})(4 \text{ s})=40$ rad. From $t=4$ s to $t=8$ s, the area is $\frac{1}{2}(-10 \text{ rad/s})(4 \text{ s})=-20$ rad. Thus, the area under the ω versus t graph during the total time interval of 8 s is 20 rad or $(20 \text{ rad})\times(1 \text{ rev}/2\pi\text{rad})=3.2$ revolutions.

4.23. Model: Treat the record on a turntable as a particle rotating at 45 rpm.
Solve: (a) The angular velocity is

$$\omega=45 \text{ rpm}\times\frac{1\text{min}}{60 \text{ s}}\times\frac{2\pi \text{ rad}}{1 \text{ rev}}=1.5\pi \text{ rad/s}$$

(b) The period is

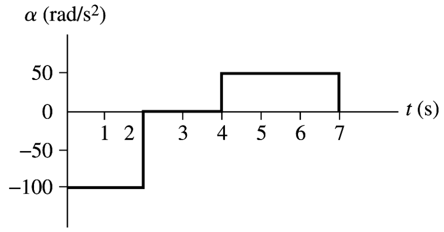
$$T=\frac{2\pi \text{ rad}}{|\omega|}=\frac{2\pi \text{ rad}}{1.5\pi \text{ rad/s}}=1.33 \text{ s}$$

4.25. Solve: Let R_E be the radius of the earth at the equator. This means R_E+300 m is the radius to the top of the tower. Letting T be the period of rotation, we have

$$v_{top}-v_{bottom}=\frac{2\pi(R_E+300 \text{ m})}{T}-\frac{2\pi R_E}{T}=\frac{2\pi(300 \text{ m})}{24 \text{ h}}=\frac{600\pi \text{ m}}{24(3600) \text{ s}}=2.18\times10^{-2} \text{ m/s}$$

4.30. Model: The crankshaft is a rotating rigid body.

Solve: The crankshaft at $t = 0$ s has an angular velocity of 250 rad/s. It gradually slows down to 50 rad/s in 2 s, maintains a constant angular velocity for 2 s until $t = 4$ s, and then speeds up to 200 rad/s from $t = 4$ s to $t = 7$ s. The angular acceleration (α) graph is based on the fact that α is the slope of the ω-versus-t graph.

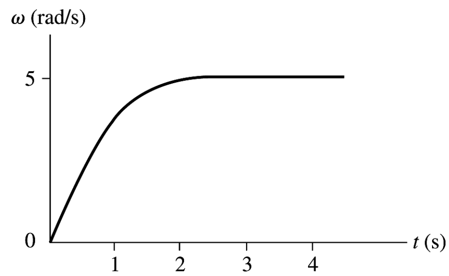


4.31. Model: The turntable is a rotating rigid body.
Solve: The angular velocity is the area under the α-versus-t graph:

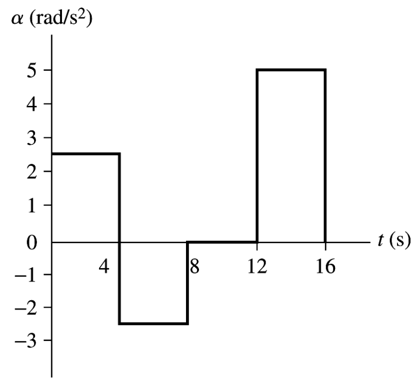
$$\alpha = \frac{d\omega}{dt} \Rightarrow \omega = \int \alpha(x)\,dt = \omega_0 + \text{area under the } \alpha \text{ graph}$$

The values of ω at selected values of time (t) are:

***t* (s)**	***ω* (rad/s)**
0	0
0.5	$(5+3.75)(0.5)/2 = 2.18$
1.0	$(5+2.5)(1)/2 = 3.75$
1.5	$(5+1.25)(1.5)/2 = 4.68$
2.0	$(5+0)(2)/2 = 5.0$
2.5	5.0
3.0	5.0



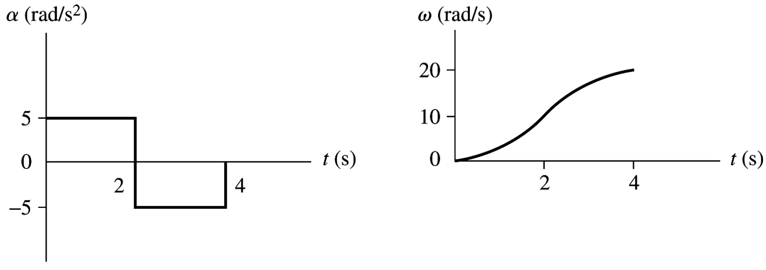
4.32. Model: The wheel is a rotating rigid body.
Solve: (a)



The angular acceleration (α) is the slope of the ω-versus-t graph.

(b) The car is at rest at $t = 0$ s. It gradually speeds up for 4 s and then slows down for 4 s. The car is at rest from $t = 8$ s to $t = 12$ s, and then speeds up again for 4 s.

4.33. Model: The angular velocity and angular acceleration graphs correspond to a rotating rigid body.
Solve:

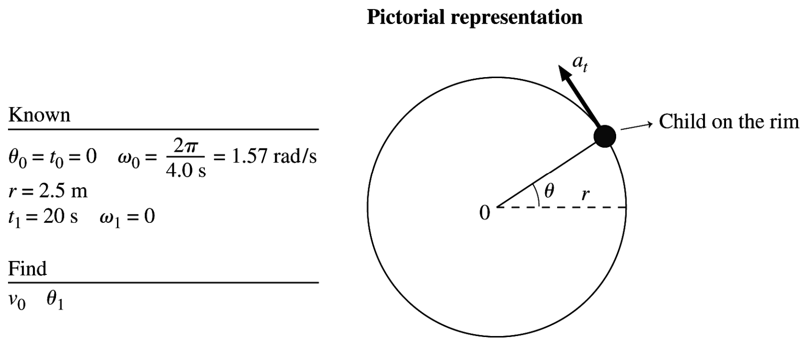


(a) (b)

(a) The α-versus-t graph has a positive slope of 5 rad/s^2 from $t = 0$ s to $t = 2$ s and a negative slope of -5 rad/s^2 from $t = 2$ s to $t = 4$ s.
(b) The angular velocity is the area under the α-versus-t graph:

$$\alpha = \frac{d\omega}{dt} \Rightarrow \omega = \int \alpha(x)\,dt = \omega_0 + \text{area under } \alpha \text{ graph.}$$

4.35. Model: Model the child on the merry-go-round as a particle in nonuniform circular motion.
Visualize:



Solve: **(a)** The speed of the child is $v_0 = r\omega = (2.5\text{ m})(1.57\text{ rad/s}) = 3.9$ m/s.
(b) The merry-go-round slows from 1.57 rad/s to 0 in 20 s. Thus

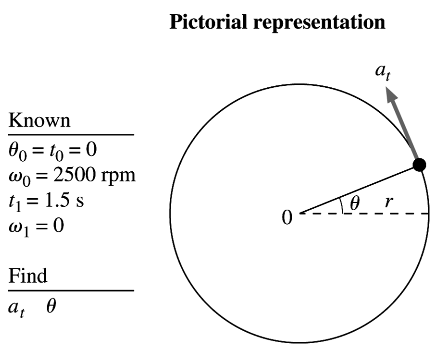
$$\omega_1 = 0 = \omega_0 + \frac{a_t}{r}t_1 \Rightarrow a_t = -\frac{r\omega_0}{t_1} = -\frac{(2.5\text{ m})(1.57\text{ rad/s})}{20\text{ s}} = -0.197\text{ m/s}^2$$

During these 20 s, the wheel turns through angle

$$\theta_1 = \theta_0 + \omega_0 t_1 + \frac{a_t}{2r}t_1^2 = 0 + (1.57\text{ rad/s})(20\text{ s}) - \frac{0.197\text{ m/s}^2}{2(2.5\text{ m})}(20\text{ s})^2 = 15.6\text{ rad}$$

In terms of revolutions, $\theta_1 = (15.6\text{ rad})(1\text{ rev}/2\pi\text{rad}) = 2.49$ rev.

4.36. Model: Model the particle on the crankshaft as being in nonuniform circular motion.
Visualize:



Solve: **(a)** The initial angular velocity is $\omega_0 = 2500\text{ rpm} \times (1\text{ min}/60\text{ s}) \times (2\pi\text{ rad/rev}) = 261.8$ rad/s. The crankshaft slows from 261.8 rad/s to 0 in 1.5 s. Thus

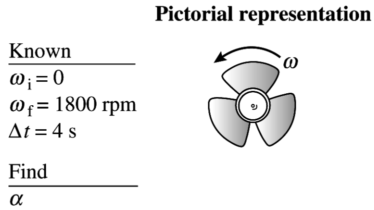
$$\omega_1 = 0 = \omega_0 + \frac{a_t}{r}t_1 \Rightarrow a_t = -\frac{r\omega_0}{t_1} = -\frac{(0.015\text{ m})(261.8\text{ rad/s})}{1.5\text{ s}} = -2.618\text{ m/s}^2 = -2.6\text{ m/s}^2$$

(b) During these 1.5 s, the crankshaft turns through angle

$$\theta_1 = \theta_0 + \omega_0 t_1 + \frac{a_t}{2r}t_1^{\,2} = 0 + (261.8\text{ rad/s})(1.5\text{ s}) - \frac{2.618\text{ m/s}^2}{2(0.015\text{ m})}(1.5\text{ s})^2 = 196\text{ rad}$$

In terms of revolutions, $\theta_1 = (196\text{ rad})(1\text{ rev}/\pi\text{rad}) = 31.2\text{ rev}$.

4.37. Model: The fan is in nonuniform circular motion.
Visualize:

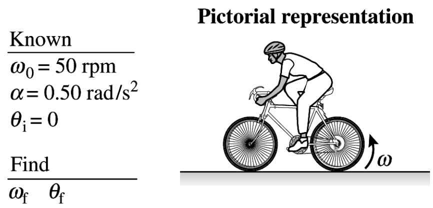


Solve: Note $1800\text{ rev/min}\left(\frac{\text{min}}{60\text{ s}}\right) = 30\text{ rev/s}$. Thus $\omega_f = \omega_i + \alpha\Delta t \Rightarrow 30\text{ rev/s} = 0\text{ rev/s} + \alpha(4.0\text{ s}) \Rightarrow \alpha = 7.5\text{ rev/s}^2$.

This can be expressed as $(7.5\text{ rev/s})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) = 47\text{ rad/s}^2$.

Assess: An increase in the angular velocity of a fan blade by 7.5 rev/s each second seems reasonable.

4.38. Model: The wheel is in nonuniform circular motion.
Visualize:



Solve: **(**a) Express ω_i in rad/s:

$$\omega_i = (50\text{ rev/min})\left(\frac{\text{min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right) \Rightarrow 5.2\text{ rad/s}$$

After 10 s, $\omega_f = \omega_i + \alpha\Delta t \Rightarrow \omega_f = 5.2\text{ rad/s} + (0.50\text{ rad/s}^2)(10\text{ s})^2 = 55\text{ rad/s}$. Converting to rpm,

$$(55\text{ rad/s})\left(\frac{60\text{ s}}{\text{min}}\right)\left(\frac{\text{rev}}{2\pi\text{ rad}}\right) = 53\text{ rpm}$$

(b) In 10 s, the wheel has turned a number of radians

$$\theta_f = \theta_i + \omega_0\Delta t + \tfrac{1}{2}\alpha\Delta t^2 \Rightarrow \theta_f = 0\text{ rad/sec} + (5.2\text{ rad/s})(10\text{ s}) + \tfrac{1}{2}(0.50\text{ rad/s}^2)(10\text{ s})^2 = 77\text{ radians.}$$

Converting, $77\text{ rad} = 12.3$ revolutions.
Assess: Making a bicycle wheel turn just over 12 revolutions in 10 s when it is initially turning almost one revolution per second to begin with seems attainable by a cyclist.