Chapter 8, Conceptual Questions

8.2. The free-body diagram (a) is correct. The forces acting on the car at the bottom of the hill are the downward gravitational force and an upward normal force. The car can be considered to be in circular motion about a point above the bottom center of the valley, which requires a net force toward the center of the circle. In this case, the circle center is above the car, so the normal force is greater than the gravitational force.

8.3. $T_c > T_a = T_d > T_b$. Use $T = \frac{mv^2}{r}$. For (a), $T_a = \frac{mv^2}{r}$. For (b), $T_b = \frac{mv^2}{2r} = \frac{1}{2}T_a$. For (c), $T_c = \frac{(2m)v^2}{r} = 2T_a$. For (d), $T_d = \frac{(2m)v^2}{2r} = T_a$.

8.4. The tension in the string at the ball's lowest point is greater than the gravitational force of the ball. The string provides the tension required to balance the gravitational force on the ball, present whether the ball is swinging or not, plus the centripetal force required to move the ball in a circle.

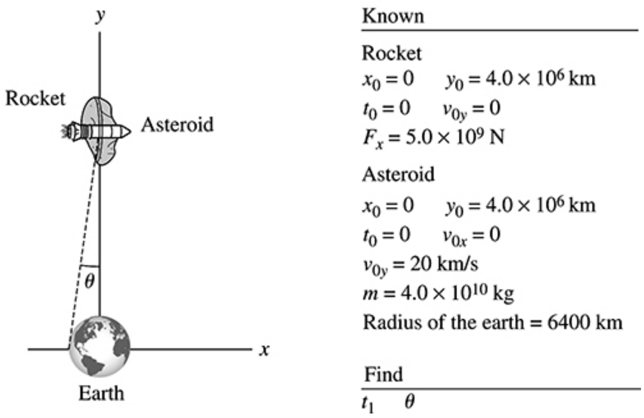
8.6. Neither Sally nor Raymond is completely correct. Both have partially correct descriptions, but are missing key points. In order to speed up, there must be a nonzero acceleration parallel to the track. In order to move in a circle, there must be a nonzero centripetal acceleration. Since both of these are required, the net force points somewhere between the forward direction (parallel to the track) and the center of the circle.

8.8. Yes, the bug is weightless because it, like the projectile it is riding in, is in free fall around the planet.

Chapter 8, Exercises and Problems

8.3. Model: The asteroid and the giant rocket will be treated as particles undergoing motion according to the constant-acceleration equations of kinematics.
Visualize:

Pictorial representation



Solve: **(a)** The time it will take the asteroid to reach the earth is

$$\frac{\text{displacement}}{\text{velocity}} = \frac{4.0\times10^6 \text{ km}}{20 \text{ km/s}} = 2.0\times10^5 \text{ s} = 56 \text{ h}$$

(b) The angle of a line that just misses the earth is

$$\tan\theta = \frac{\text{R}}{y_0} \Rightarrow \theta = \tan^{-1}\left(\frac{\text{R}}{y_0}\right) = \tan^{-1}\left(\frac{6400 \text{ km}}{4.0\times10^6 \text{ km}}\right) = 0.092°$$

(c) When the rocket is fired, the horizontal acceleration of the asteroid is

$$a_x = \frac{5.0\times10^9 \text{ N}}{4.0\times10^{10} \text{ kg}} = 0.125 \text{ m/s}^2$$

(Note that the mass of the rocket is much smaller than the mass of the asteroid and can therefore be ignored completely.) The velocity of the asteroid after the rocket has been fired for 300 s is

$$v_x = v_{0x} + a_x(t - t_0) = 0 \text{ m/s} + \left(0.125 \text{ m/s}^2\right)\left(300 \text{ s} - 0 \text{ s}\right) = 37.5 \text{ m/s}$$

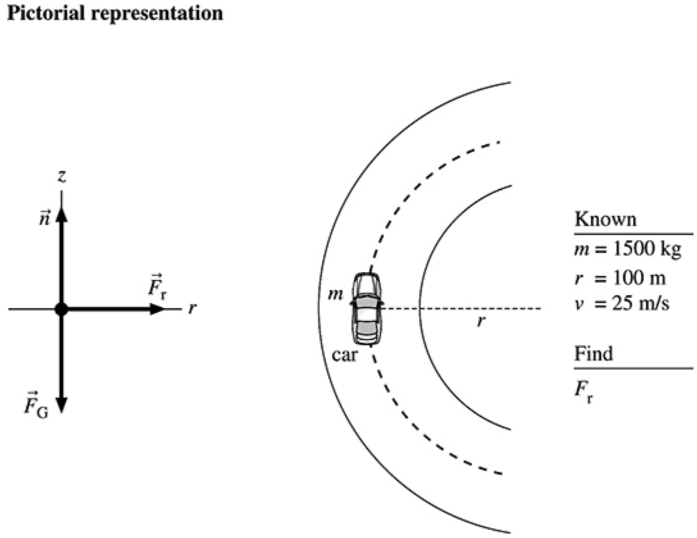
After 300 s, the vertical velocity is $v_y = 2\times10^4$ m/s and the horizontal velocity is $v_x = 37.5$ m/s. The deflection due to this horizontal velocity is

$$\tan\theta = \frac{v_x}{v_y} \Rightarrow \theta = \tan^{-1}\left(\frac{37.5 \text{ m/s}}{2\times10^4 \text{ m/s}}\right) = 0.107°$$

That is, the earth is saved.

8.4. Model: We are using the particle model for the car in uniform circular motion on a flat circular track. There must be friction between the tires and the road for the car to move in a circle.

Visualize:



Solve: The centripetal acceleration is

$$a_r = \frac{v^2}{r} = \frac{(25\text{ m/s})^2}{100\text{ m}} = 6.25\text{ m/s}^2$$

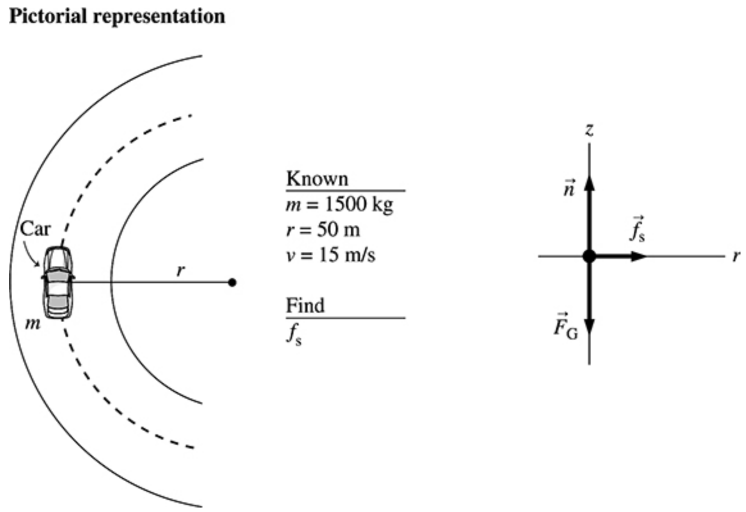
The acceleration points to the center of the circle, so the net force is

$$\vec{F}_r = m\vec{a} = (1500\text{ kg})(6.25\text{ m/s}^2,\text{ toward center})$$
$$= (9380\text{ N, toward center})$$

This force is provided by static friction

$$f_s = F_r = 9.4\text{ kN}$$

8.5. Model: We will use the particle model for the car which is in uniform circular motion.
Visualize:



Solve: The centripetal acceleration of the car is

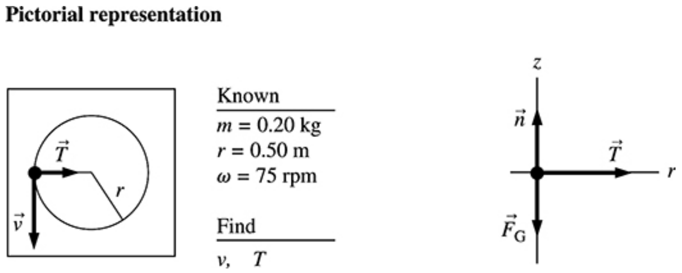
$$a_r = \frac{v^2}{r} = \frac{(15\text{ m/s})^2}{50\text{ m}} = 4.5\text{ m/s}^2$$

The acceleration is due to the force of static friction. The force of friction is $f_s = ma_r = (1500\text{ kg})(4.5\text{m/s}^2) = 6750\text{ N} = 6.8\text{ kN}$.

Assess: The model of static friction is $(f_s)_{max} = n\mu_s = mg\mu_s \approx mg \approx 15{,}000\text{ N}$ since $\mu_s \approx 1$ for a dry road surface. We see that $f_s < (f_s)_{max}$, which is reasonable.

8.6. Model: Treat the block as a particle attached to a massless string that is swinging in a circle on a frictionless table.

Visualize:



Solve: **(a)** The angular velocity and speed are

$$\omega = 75\frac{\text{rev}}{\text{min}}\times\frac{2\pi\ \text{rad}}{1\ \text{rev}} = 471.2\ \text{rad/min} \quad v_t = r\omega = (0.50\ \text{m})(471.2\ \text{rad/min})\times\frac{1\text{min}}{60\ \text{s}} = 3.93\ \text{m/s}$$

The tangential velocity is 3.9 m/s.

(b) The radial component of Newton's second law is

$$\sum F_r = T = \frac{mv^2}{r}$$

Thus

$$T = (0.20\ \text{kg})\frac{(3.93\ \text{m/s})^2}{0.50\ \text{m}} = 6.2\ \text{N}$$

8.7. Solve: Newton's second law is $F_r = ma_r = mr\omega^2$. Substituting into this equation yields:

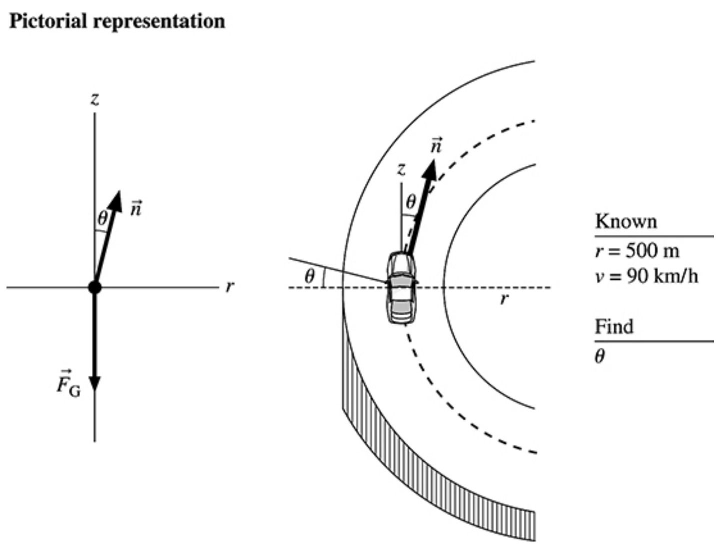
$$\omega = \sqrt{\frac{F_r}{mr}} = \sqrt{\frac{8.2\times10^{-8}\ \text{N}}{(9.1\times10^{-31}\ \text{kg})(5.3\times10^{-11}\ \text{m})}}$$

$$= 4.37\times10^{16}\ \text{rad/s} = 4.37\times10^{16}\frac{\text{rad}}{\text{s}}\times\frac{1\ \text{rev}}{2\pi\ \text{rad}} = 6.6\times10^{15}\ \text{rev/s}$$

Assess: This is a very high number of revolutions per second.

8.8. Model: The vehicle is to be treated as a particle in uniform circular motion.

Visualize:



On a banked road, the normal force on a vehicle has a horizontal component that provides the necessary centripetal acceleration. The vertical component of the normal force balances the gravitational force.

Solve: From the physical representation of the forces in the *r-z* plane, Newton's second law can be written

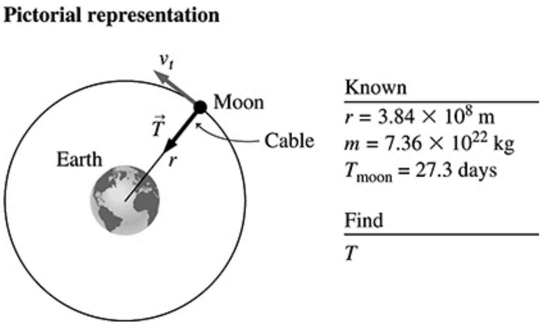
$$\sum F_r = n\sin\theta = \frac{mv^2}{r} \quad \sum F_z = n\cos\theta - mg = 0 \Rightarrow n\cos\theta = mg$$

Dividing the two equations and making the conversion $90\text{ km/h} = 25\text{ m/s}$ yields:

$$\tan\theta = \frac{v^2}{rg} = \frac{(25\text{ m/s})^2}{(9.8\text{ m/s}^2)500\text{ m}} = 0.128 \Rightarrow \theta = 7.3°$$

Assess: Such a banking angle for a speed of approximately 55 mph is clearly reasonable and within our experience as well.

8.9. Model: The motion of the moon around the earth will be treated through the particle model. The circular motion is uniform.
Visualize:



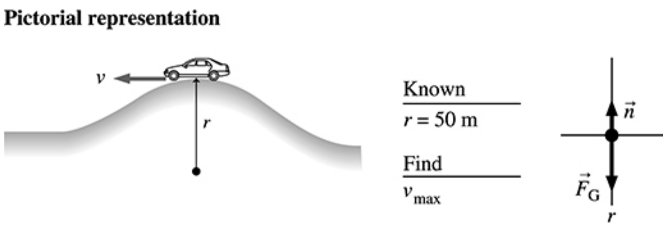
Solve: The tension in the cable provides the centripetal acceleration. Newton's second law is

$$\sum F_r = T = mr\omega^2 = mr\left(\frac{2\pi}{T_{\text{moon}}}\right)^2$$

$$= (7.36\times10^{22}\text{ kg})(3.84\times10^{8}\text{ m})\left[\frac{2\pi}{27.3\text{ days}}\times\frac{1\text{ day}}{24\text{ h}}\times\frac{1\text{ h}}{3600\text{ s}}\right]^2 = 2.01\times10^{20}\text{ N}$$

Assess: This is a tremendous tension, but clearly understandable in view of the moon's large mass and the large radius of circular motion around the earth.

8.13. Model: Use the particle model for the car which is undergoing circular motion.
Visualize:



Solve: The car is in circular motion with the center of the circle below the car. Newton's second law at the top of the hill is

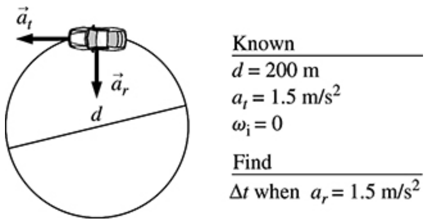
$$\sum F_r = (F_G)_r - n_r = mg - n = ma_r = \frac{mv^2}{r} \Rightarrow v^2 = r\left(g - \frac{n}{m}\right)$$

Maximum speed is reached when $n = 0$ and the car is beginning to lose contact with the road.

$$v_{\text{max}} = \sqrt{rg} = \sqrt{(50\text{ m})(9.8\text{ m/s}^2)} = 22\text{ m/s}$$

Assess: A speed of 22 m/s is equivalent to 49 mph, which seems like a reasonable value.

8.18. Model: Use the particle model for the car, which is undergoing nonuniform circular motion.
Visualize:



Solve: The car is in circular motion with radius $r = \frac{d}{2} = 100 \text{ m}$. We require

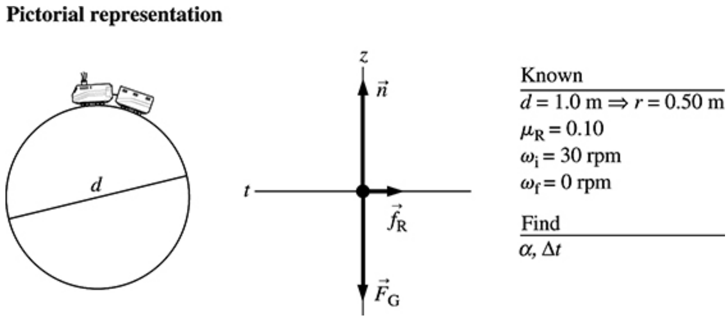
$$a_r = \omega^2 r = 1.5 \text{ m/s}^2 \Rightarrow \omega = \sqrt{\frac{1.5 \text{ m/s}^2}{r}} = \sqrt{\frac{1.5 \text{ m/s}^2}{100 \text{ m}}} = 0.122 \text{ s}^{-1}$$

The definition of the angular velocity can be used to determine the time Δt using the angular acceleration $\alpha = \frac{a_t}{r} = \frac{1.5 \text{ m/s}^2}{100 \text{ m}} = 1.5\times10^{-2} \text{ s}^{-2}$.

$$\omega = \omega_i + \alpha\Delta t$$

$$\Rightarrow \Delta t = \frac{\omega - \omega_i}{\alpha} = \frac{0.122 \text{ s}^{-1} - 0 \text{ s}^{-1}}{0.015 \text{ s}^{-2}} = 8.2 \text{ s}$$

8.19. Model: The train is a particle undergoing nonuniform circular motion.
Visualize:



Solve: **(a)** Newton's second law in the vertical direction is

$$(F_{\text{net}})_y = n - F_G = 0$$

from which $n = mg$. The rolling friction is $f_R = \mu_R n = \mu_R mg$. This force provides the tangential acceleration

$$a_t = -\frac{f_R}{m} = -\mu_R g$$

The angular acceleration is

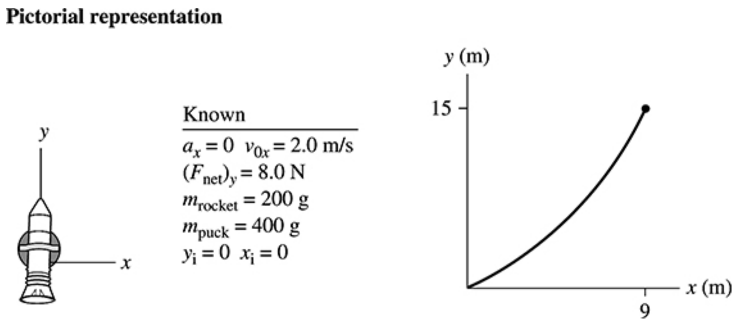
$$\alpha = \frac{a_t}{r} = \frac{-\mu_R g}{r} = \frac{-(0.10)(9.8 \text{ m/s}^2)}{0.50 \text{ m}} = -1.96 \text{ rad/s}^2$$

(b) The initial angular velocity is $30\left(\frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right)\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 3.14 \text{ rad/s}$. The time to come to a stop due to the rolling friction is

$$\Delta t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 3.14 \text{ rad/s}}{-1.96 \text{ rad/s}^2} = 1.60 \text{ s}$$

Assess: The original angular speed of π rad/s means the train goes around the track one time every 2 seconds, so a stopping time of less than 2 s is reasonable.

8.21. Model: The rocket and puck together make a particle moving on frictionless ice. The thrust of the rocket motor is assumed to be constant.
Visualize:



Solve The kinematics equations can be used to examine the motion for each coordinate axis independently as a function of time *t*, then combined to eliminate *t*.
In the *x*-direction,

$$x_f = x_i + v_{0x}t = 0\text{ m} + (2.0\text{ m/s})t$$

So $t = \dfrac{x_f}{2.0\text{ m/s}}$. The acceleration of the rocket and puck in the y-direction is

$$a_y = \frac{(F_{net})y}{m_{\text{rocket}} + m_{\text{puck}}} = \frac{8.0\text{ N}}{0.600\text{ kg}} = 13.3\text{ m/s}^2$$

In the y–direction,

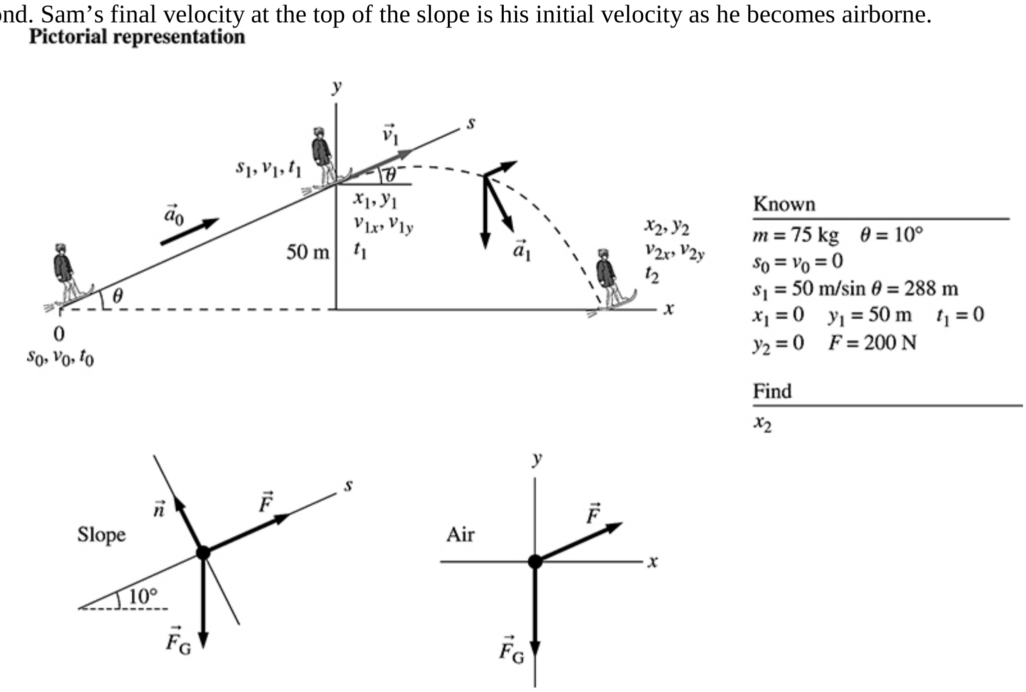
$$\begin{aligned} y_\text{f} &= y_\text{i} + v_{0y}t + \frac{1}{2}a_y t^2 = 0\text{ m} + (0\text{ m/s})t + \frac{1}{2}(13.3\text{ m/s}^2)t^2 \\ &= (6.67\text{ m/s}^2)t^2 \end{aligned}$$

Substituting t from the equation for the x-direction,

$$y_\text{f} = (6.67\text{ m/s}^2)\left(\frac{x}{2.0\text{ m/s}}\right)^2 = 1.67x_\text{f}^2$$

A graph of this equation is shown in the figure.
Assess: When the puck has traveled 9 m in the x-direction it has traveled 15 m in the y-direction.

8.22. Model: Treat Sam as a particle.
Visualize: This is a two-part problem. Use an s-axis parallel to the slope for the first part, regular xy-coordinates for the second. Sam's final velocity at the top of the slope is his initial velocity as he becomes airborne.



Solve: Sam's acceleration up the slope is given by Newton's second law:

$$\begin{aligned} (F_{\text{net}})_s &= F - mg\sin 10° = ma_0 \\ a_0 &= \frac{F}{m} - g\sin 10° = \frac{200\text{ N}}{75\text{ kg}} - (9.8\text{ m/s}^2)\sin 10° = 0.965\text{ m/s}^2 \end{aligned}$$

The length of the slope is $s_1 = (50\text{ m})/\sin 10° = 288\text{ m}$. His velocity at the top of the slope is

$$v_1^2 = v_0^2 + 2a_0(s_1 - s_0) = 2a_0 s_1 \Rightarrow v_1 = \sqrt{2(0.965\text{ m/s}^2)(288\text{ m})} = 23.6\text{ m/s}$$

This is Sam's initial speed into the air, giving him velocity components $v_{1x} = v_1\cos 10° = 23.2\text{ m/s}$ and $v_{1y} = v_1\sin 10° = 410\text{ m/s}$. This is not projectile motion because Sam experiences both the force of gravity *and* the thrust of his skis. Newton's second law for Sam's acceleration is

$$a_{1x} = \frac{(F_{net})_x}{m} = \frac{(200\text{ N})\cos 10°}{75\text{ kg}} = 2.63\text{ m/s}^2$$

$$a_{1y} = \frac{(F_{net})_y}{m} = \frac{(200\text{ N})\sin 10° - (75\text{ kg})(9.80\text{ m/s}^2)}{75\text{ kg}} = -9.34\text{ m/s}^2$$

The *y*-equation of motion allows us to find out how long it takes Sam to reach the ground:

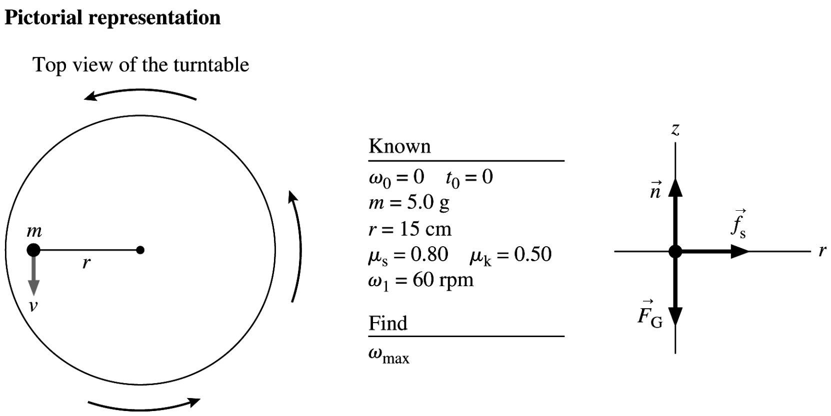
$$y_2 = 0\text{ m} = y_1 + v_{1y}t_2 + \tfrac{1}{2}a_{1y}t_2^2 = 50\text{ m} + (4.10\text{ m/s})\,t_2 - (4.67\text{ m/s}^2)\,t_2^2$$

This quadratic equation has roots $t_2 = -2.86$ s (unphysical) and $t_2 = 3.74$ s. The *x*-equation of motion—this time with an acceleration—is

$$x_2 = x_1 + v_{1x}t_2 + \tfrac{1}{2}a_{1x}t_2^2 = 0\text{ m} + (23.2\text{ m/s})\,t_2 - \tfrac{1}{2}(2.63\text{ m/s}^2)\,t_2^2 = 105\text{ m}$$

Sam lands 105 m from the base of the cliff.

8.33. Model: Use the particle model and static friction model for the coin, which is undergoing circular motion.
Visualize:



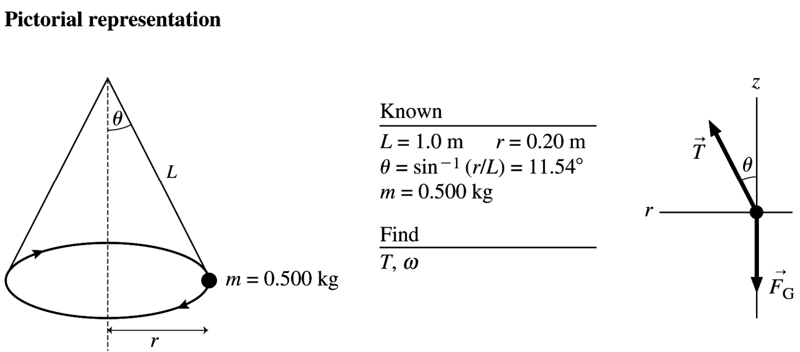
Solve: The force of static friction is $f_s = \mu_s n = \mu_s mg$. This force is equivalent to the maximum centripetal force that can be applied without sliding. That is,

$$\mu_s mg = m\frac{v_t^2}{r} = m\left(r\omega_{max}^2\right) \Rightarrow \omega_{max} = \sqrt{\frac{\mu_s g}{r}} = \sqrt{\frac{(0.80)(9.8\text{ m/s}^2)}{0.15\text{ m}}} = 7.23\text{ rad/s}$$

$$= 7.23\ \frac{\text{rad}}{\text{s}} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 69\text{ rpm}$$

So, the coin will stay still on the turntable.
Assess: A rotational speed of approximately 1 rev per second for the coin to stay stationary seems reasonable.

8.35. Model: Use the particle model for the ball in circular motion.
Visualize:



Solve: **(a)** The mass moves in a *horizontal* circle of radius $r = 20$ cm. The acceleration $\vec{a}$ and the net force vector point to the center of the circle, *not* along the string. The only two forces are the string tension $\vec{T}$, which does point along the string, and the gravitational force $\vec{F}_G$. These are shown in the free-body diagram. Newton's second law for circular motion is

$$\sum F_z = T\cos\theta - F_G = T\cos\theta - mg = 0\text{ N} \quad \sum F_r = T\sin\theta = ma_r = \frac{mv^2}{r}$$

From the *z*-equation,

$$T = \frac{mg}{\cos\theta} = \frac{(0.500\text{ kg})(9.8\text{ m/s}^2)}{\cos 11.54°} = 5.00\text{ N}$$

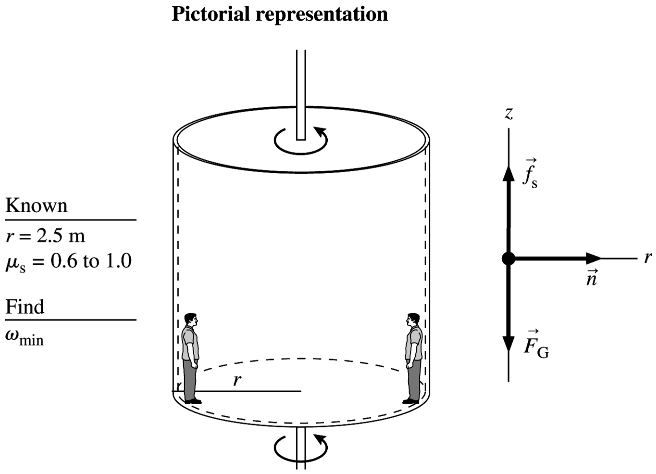
(b) We can find the tangential speed from the *r*-equation:

$$v = \sqrt{\frac{rT\sin\theta}{m}} = 0.63\text{ m/s}$$

The angular speed is

$$\omega = \frac{v}{r} = \frac{0.63\text{ ms}}{0.20\text{ m}} = 3.15\text{ rad/s} = 30\text{ rpm}$$

8.36. Model: Consider the passenger to be a particle and use the model of static friction.
Visualize:



Solve: The passengers stick to the wall if the static friction force is sufficient to support the gravitational force on them: $f_s = F_G$ The minimum angular velocity occurs when static friction reaches its maximum possible value $(f_s)_{max} = \mu_s n.$ Although clothing has a range of coefficients of friction, it is the clothing with the smallest coefficient $(\mu_s = 0.60)$ that will slip first, so this is the case we need to examine. Assuming that the person is stuck to the wall, Newton's second law is

$$\sum F_r = n = m\omega^2 r \qquad \sum F_z = f_s - w = 0 \Rightarrow f_s = mg$$

The minimum frequency occurs when

$$f_s = (f_s)_{max} = \mu_s n = \mu_s m r \omega_{min}^2$$

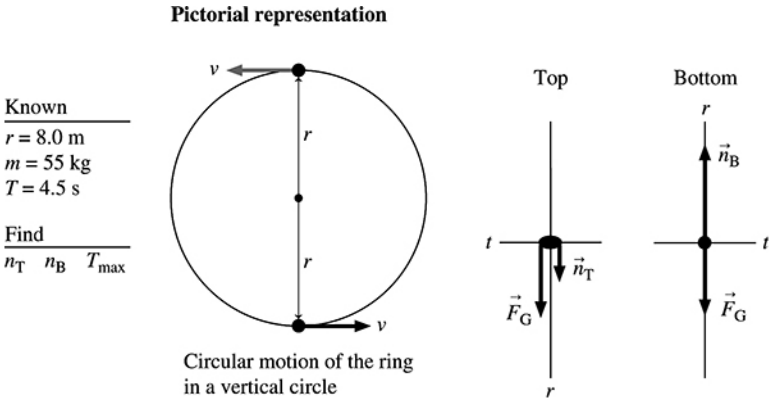
Using this expression for f_s in the *z*-equation gives

$$f_s = \mu_s m r \omega_{min}^2 = mg$$

$$\Rightarrow \omega_{min} = \sqrt{\frac{g}{\mu_s r}} = \sqrt{\frac{9.80\text{ m/s}^2}{0.60(2.5\text{ m})}} = 2.56\text{ rad/s} = 2.56\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}} = 24\text{ rpm}$$

Assess: Note the velocity does not depend on the mass of the individual. Therefore, the minimum mass sign is not necessary.

8.41. Model: Model a passenger as a particle rotating in a vertical circle.
Visualize:



Solve: **(a)** Newton's second law at the top is

$$\sum F_r = n_T + F_G = ma_r = \frac{mv^2}{r} \Rightarrow n_T + mg = \frac{mv^2}{r}$$

The speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi(8.0 \text{ m})}{4.5 \text{ s}} = 11.17 \text{ m/s}$$

$$\Rightarrow n_T = m\left(\frac{v^2}{r} - g\right) = (55 \text{ kg})\left[\frac{(11.17 \text{ m/s})^2}{8.0 \text{ m}} - 9.8 \text{ m/s}^2\right] = 319 \text{ N}$$

That is, the ring pushes on the passenger with a force of 3.2×10^2 N at the top of the ride. Newton's second law at the bottom:

$$\sum F_r = n_B - F_G = ma_r = \frac{mv^2}{r} \Rightarrow n_B = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right)$$

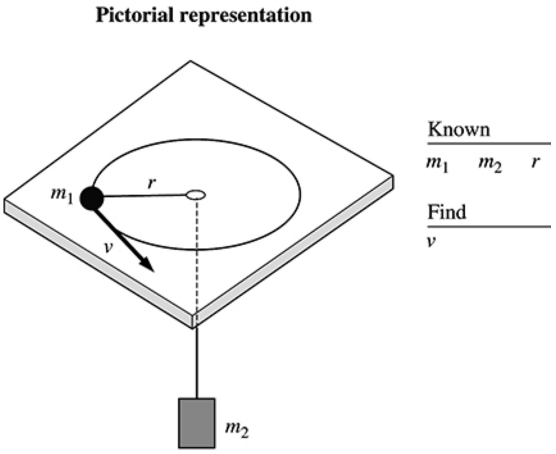
$$= (55 \text{ kg})\left[\frac{(11.17 \text{ m/s})^2}{8.0 \text{ m}} + 9.8 \text{ m/s}^2\right] = 1397 \text{ N}$$

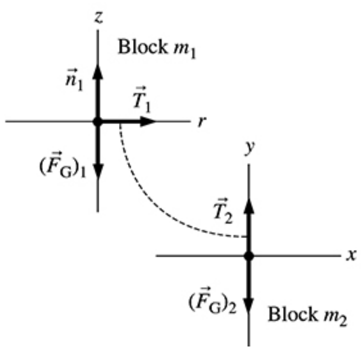
Thus the force with which the ring pushes on the rider when she is at the bottom of the ring is 1.4 kN.
(b) To just stay on at the top, $n_T = 0$ N in the r-equation at the top in part (a). Thus,

$$mg = \frac{mv^2}{r} = mr\omega^2 = mr\left(\frac{2\pi}{T_{max}}\right)^2 \Rightarrow T_{max} = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{8.0 \text{ m}}{9.8 \text{ m/s}^2}} = 5.7 \text{ s}$$

8.46. Model: Masses m_1 and m_2 are considered particles. The string is assumed to be massless.

Visualize:





Solve: The tension in the string causes the centripetal acceleration of the circular motion. If the hole is smooth, it acts like a pulley. Thus tension forces $\vec{T}_1$ and $\vec{T}_2$ act as if they were an action/reaction pair. Mass m_1 is in circular motion of radius *r*, so Newton's second law for m_1 is

$$\sum F_r = T_1 = \frac{m_1 v^2}{r}$$

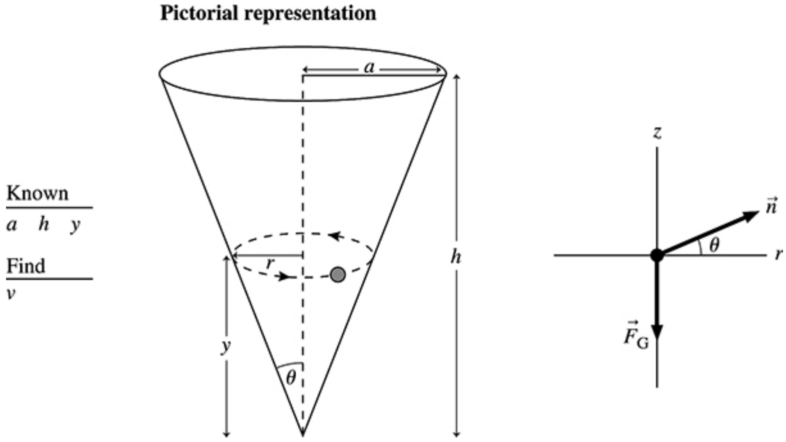
Mass m_2 is at rest, so the *y*-equation of Newton's second law is

$$\sum F_y = T_2 - m_2 g = 0 \text{ N} \Rightarrow T_2 = m_2 g$$

Newton's third law tells us that $T_1 = T_2$. Equating the two expressions for these quantities:

$$\frac{m_1 v^2}{r} = m_2 g \Rightarrow v = \sqrt{\frac{m_2 r g}{m_1}}$$

8.58. Model: Use the particle model for the ball, which is in uniform circular motion.
Visualize:



Solve: From Newton's second law along *r* and *z* directions,

$$\sum F_r = n\cos\theta = \frac{mv^2}{r} \quad \sum F_z = n\sin\theta - mg = 0 \Rightarrow n\sin\theta = mg$$

Dividing the two force equations gives

$$\tan\theta = \frac{gr}{v^2}$$

From the geometry of the cone, $\tan\theta = r/y$. Thus

$$\frac{r}{y} = \frac{gr}{v^2} \Rightarrow v = \sqrt{gy}$$