

## Chapter 9, Conceptual Questions

**9.1.** The velocities and masses vary from object to object, so there is no choice but to compute  $p_x = mv_x$  for each one and then compare.

$$p_{1,x} = (20 \text{ g})(1 \text{ m/s}) = 20 \text{ g} \cdot \text{m/s}$$

$$p_{2,x} = (20 \text{ g})(2 \text{ m/s}) = 40 \text{ g} \cdot \text{m/s}$$

$$p_{3,x} = (10 \text{ g})(2 \text{ m/s}) = 20 \text{ g} \cdot \text{m/s}$$

$$p_{4,x} = (10 \text{ g})(1 \text{ m/s}) = 10 \text{ g} \cdot \text{m/s}$$

$$p_{5,x} = (200 \text{ g})(0.1 \text{ m/s}) = 20 \text{ g} \cdot \text{m/s}$$

So the answer is  $p_{2,x} > p_{1,x} = p_{3,x} = p_{5,x} > p_{4,x}$ .

**9.4. Reason:** When the question talks about forces, times, and momenta, we immediately think of the impulse-momentum theorem, which tells us that to change the momentum of an object we must exert a net external force on it over a time interval:  $\vec{F}_{\text{avg}} \Delta t = \Delta \vec{p}$ .

Because equal forces are exerted over equal times, the impulses are equal and the changes in momentum are equal. Because both carts start from rest, their changes in momentum are the same as the final momentum for each, so their final momenta are equal. Notice that we did not need to know the mass of either cart, or even the specific time interval (as long as it was the same for both carts) to answer the question.

**9.5.** The impulse-momentum theory tells us that the change in momentum of an object is related to the net force on the object and the length of time the force was applied. Mathematically,  $\Delta \vec{p} = \vec{F}_{\text{avg}} \Delta t$ . The same force applied to the two carts results in a larger acceleration for the less massive plastic cart, enabling it to travel the 1 m distance in a shorter time. Therefore the plastic cart has a smaller change in momentum (which is the same as the final momentum since it started at rest) than the lead cart.

**9.6.** In this story, Carlos is correct. During the short time of the bullet-block collision other forces are negligible compared to the force between the bullet and block, so in the impulse approximation momentum is conserved. In the case in which the bullet bounces off of the steel block, the bullet's final momentum is backward. To balance that, the steel block must be moving forward faster than the case in which the bullet embeds itself in the wooden block and has a final forward momentum.

**9.7.** The impulse-momentum theory states that a change in an object's momentum results when a net force is applied to the object for some time interval,  $\Delta p_x = (F_{\text{avg}})_x \Delta t$ . Stopping a hard ball requires changing its momentum from some to none. That change can be accomplished with a small force over a long time interval or a large force over a short time interval. The padding in a glove lets the time interval during which the ball is stopped be long, resulting in a smaller force on the glove and your hand.

**9.8.** The impulse-momentum theory states that a change in an object's momentum results when a net force is applied to the object for some time interval,  $\Delta p_x = (F_{\text{avg}})_x \Delta t$ . Stopping an automobile requires changing its momentum from some to none. That change can be accomplished with a small force over a long time interval or a large force over a short time interval. The crumple zone that collapses during an automobile collision lets the time interval during which the automobile is stopped be long, resulting in a smaller force on the passengers as they also come to a stop.

**9.9.** The impulse is equal to the change in momentum, so

$$\Delta p_x = mv_{\text{fx}} - mv_{\text{ix}} = 4 \text{ Ns}$$

The final velocity is thus

$$v_{\text{fx}} = v_{\text{ix}} + \frac{4 \text{ Ns}}{m} = 1 \text{ m/s} + \frac{4 \text{ Ns}}{2 \text{ kg}} = 3 \text{ m/s}$$

Since the velocity is positive, the object is moving to the right.

**9.10.** The impulse is equal to the change in momentum, so

$$\Delta p_x = mv_{\text{fx}} - mv_{\text{ix}} = -4 \text{ Ns}$$

The final velocity is thus

$$v_{\text{fx}} = v_{\text{ix}} - \frac{4 \text{ Ns}}{m} = 1 \text{ m/s} - \frac{4 \text{ Ns}}{2 \text{ kg}} = -1 \text{ m/s}$$

Since the velocity is negative, the object is now moving to the left. Note that the impulse was negative, which decreases the initial positive velocity.

**9.11.** The club and ball form a system. The interaction force when the club and ball collide is very large compared to other forces at the time of collision, such as gravity and the force of the golfer on the club. So, in this impulse approximation, momentum is conserved during the collision. After the club hits the ball, it will give the ball some of its momentum. The club can continue to move forward as long as the momentum the ball obtains is less than the initial momentum of the club.

Note that the momentum conservation is only valid if we consider the short time between just before and just after the collision. The wider we make the time window, the more time gravity and the golfer have to influence the motion of the club and ball, so that momentum conservation would not hold.

**9.12.** The impulse one ball receives is equal to the average force on it from the other ball multiplied by the time during which the force is applied. But by Newton's third law the force that the rubber ball exerts on the steel ball is equal to the force the steel ball exerts on the rubber ball. So both balls receive the same amount of impulse, although the impulses are in opposite directions.

**9.13. (a)** Both particles cannot be at rest immediately after the collision. If they were both at rest, then the sum of the momenta after the collision would be zero, and since momentum is conserved in collisions, it would have had to be zero before as well (and it wasn't).

**(b)** If the masses are equal and the collision elastic, the moving particle will stop and give all of its momentum to the previously resting particle. A good example of this appears when a billiard ball rolls directly into another resting billiard ball.

We say momentum is conserved in all collisions because we assume that both colliding objects are part of the system and we assume the "impulse approximation" that other forces can be neglected during the short time interval of the collision.

In part (a) if the system contained a third particle that participated in the collision, then it is possible for the first two particles to end up at rest if the momentum is carried off by the third.

**9.14. (a)** Let Paula and Ricardo be a system. Initially, their total momentum is zero since they are at rest. After they push off each other, the total momentum must still be zero, so Ricardo and Paula must have equal but opposite momenta.

**(b)** Momentum  $p = mv$ . Since Paula is less massive than Ricardo, her speed must be higher than Ricardo's in order for her to have the same momentum as Ricardo.

**9.15.** Since the object is initially at rest, its total momentum is zero. After it explodes the total momentum of the fragments must also be zero. With  $\vec{p}_1 = (-2, 2)$  kg m/s and  $\vec{p}_2 = (3, 0)$  kg m/s, the requirement that  $\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$  means that  $\vec{p}_3 = (-1, -2)$  kg m/s.

## Chapter 9, Exercises and Problems

**9.1. Model:** Model the car and the baseball as particles.

**Solve:** (a) The momentum  $p = mv = (1500 \text{ kg})(10 \text{ m/s}) = 1.5 \times 10^4 \text{ kg m/s}$ .

(b) The momentum  $p = mv = (0.2 \text{ kg})(40 \text{ m/s}) = 8.0 \text{ kg m/s}$ .

**9.3. Visualize:** Please refer to Figure EX9.3.

**Solve:** The impulse  $J_x$  is defined in Equation 9.6 as

$$J_x = \int_{t_i}^{t_f} F_x(t) dt = \text{area under the } F_x(t) \text{ curve between } t_i \text{ and } t_f$$

$$J_x = \frac{1}{2}(4 \text{ ms})(1000 \text{ N}) + (6 - 4 \text{ ms})(1000 \text{ N}) = 4 \text{ N s}$$

**9.5. Visualize:** Please refer to Figure EX9.5.

**Solve:** The impulse is defined in Equation 9.6 as

$$J_x = \int_{t_i}^{t_f} F_x(t) dt = \text{area under the } F_x(t) \text{ curve between } t_i \text{ and } t_f$$

$$\Rightarrow 6.0 \text{ N s} = \frac{1}{2}(F_{\text{max}})(8 \text{ ms}) \Rightarrow F_{\text{max}} = 1.5 \times 10^3 \text{ N}$$

**9.7. Model:** Model the object as a particle and the interaction with the force as a collision.

**Visualize:** Please refer to Figure EX9.7.

**Solve:** Using the equations

$$p_{\text{fx}} = p_{\text{ix}} + J_x \text{ and } J_x = \int_{t_i}^{t_f} F_x(t) dt = \text{area under force curve}$$

$$(2.0 \text{ kg})v_{\text{fx}} = (2.0 \text{ kg})(1.0 \text{ m/s}) + (\text{area under the force curve})$$

$$\Rightarrow v_{\text{fx}} = (1.0 \text{ m/s}) + \frac{1}{2.0 \text{ kg}}(1.0 \text{ s})(2.0 \text{ N}) = 2.0 \text{ m/s}$$

Because  $v_{\text{fx}}$  is positive, the object moves to the right at 2.0 m/s.

**Assess:** For an object with positive velocity, a positive impulse increases the object's speed. The opposite is true for an object with negative velocity.

**9.8. Model:** Model the object as a particle and the interaction with the force as a collision.

**Visualize:** Please refer to Figure EX9.8.

**Solve:** Using the equations

$$p_{\text{fx}} = p_{\text{ix}} + J_x \text{ and } J_x = \int_{t_i}^{t_f} F_x(t) dt = \text{area under force curve}$$

$$(2.0 \text{ kg})v_{\text{fx}} = (2.0 \text{ kg})(1.0 \text{ m/s}) + (\text{area under the force curve})$$

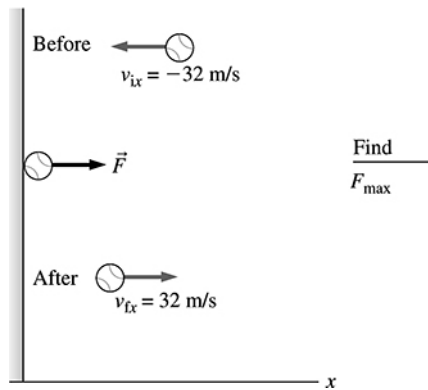
$$\Rightarrow v_{\text{fx}} = (1.0 \text{ m/s}) + \left(\frac{1}{2.0 \text{ kg}}\right)(-2.0 \text{ N})(0.50 \text{ s}) = 0.50 \text{ m/s}$$

**Assess:** For an object with positive velocity, a negative impulse slows the object. The opposite is true for an object with negative velocity.

**9.11. Model:** Model the tennis ball as a particle, and its interaction with the wall as a collision.

**Visualize:**

**Pictorial representation**



The force increases to  $F_{\max}$  during the first two ms, stays at  $F_{\max}$  for two ms, and then decreases to zero during the last two ms. The graph shows that  $F_x$  is positive, so the force acts to the right.

**Solve:** Using the impulse-momentum theorem  $p_{fx} = p_{ix} + J_x$ ,

$$(0.06 \text{ kg})(32 \text{ m/s}) = (0.06 \text{ kg})(-32 \text{ m/s}) + \int_0^{6 \text{ ms}} F_x(t) dt$$

The impulse is

$$\int_0^{6 \text{ ms}} F_x(t) dx = \text{area under force curve} = \frac{1}{2} F_{\max} (0.002 \text{ s}) + F_{\max} (0.002 \text{ s}) + \frac{1}{2} F_{\max} (0.002 \text{ s}) = (0.004 \text{ s}) F_{\max}$$

$$\Rightarrow F_{\max} = \frac{(0.06 \text{ kg})(32 \text{ m/s}) + (0.06 \text{ kg})(32 \text{ m/s})}{0.004 \text{ s}} = 9.6 \times 10^2 \text{ N}$$

**9.12. Model:** Model the ball as a particle, and its interaction with the wall as a collision in the impulse approximation.

**Visualize:** Please refer to Figure EX9.12.

**Solve:** Using the equations

$$p_{fx} = p_{ix} + J_x \text{ and } J_x = \int_{t_i}^{t_f} F_x(t) dt = \text{area under force curve}$$

$$(0.250 \text{ kg})v_{fx} = (0.250 \text{ kg})(-10 \text{ m/s}) + (500 \text{ N})(8.0 \text{ ms})$$

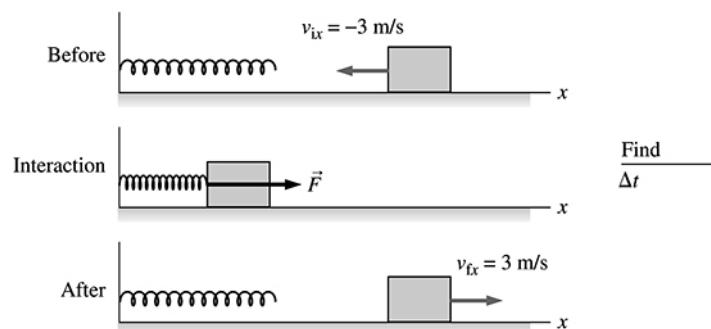
$$\Rightarrow v_{fx} = (-10 \text{ m/s}) + \left( \frac{4.0 \text{ N}}{0.250 \text{ kg}} \right) = 6 \text{ m/s}$$

**Assess:** The ball's final velocity is positive, indicating it has turned around.

**9.13. Model:** Model the glider cart as a particle, and its interaction with the spring as a collision.

**Visualize:**

**Pictorial representation**

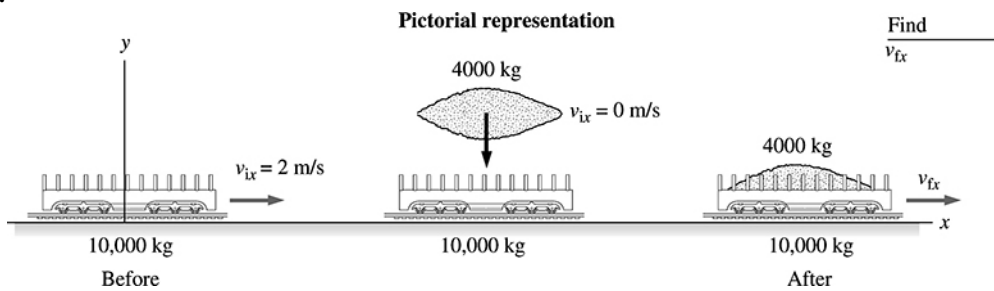


**Solve:** Using the impulse-momentum theorem  $p_{fx} - p_{ix} = \int F dt$ ,

$$(0.6 \text{ kg})(3 \text{ m/s}) - (0.6 \text{ kg})(-3 \text{ m/s}) = \text{area under force curve} = \frac{1}{2}(36 \text{ N})(\Delta t) \Rightarrow \Delta t = 0.20 \text{ s}$$

**9.14. Model:** Choose car + gravel to be the system. Ignore friction in the impulse approximation.

**Visualize:**

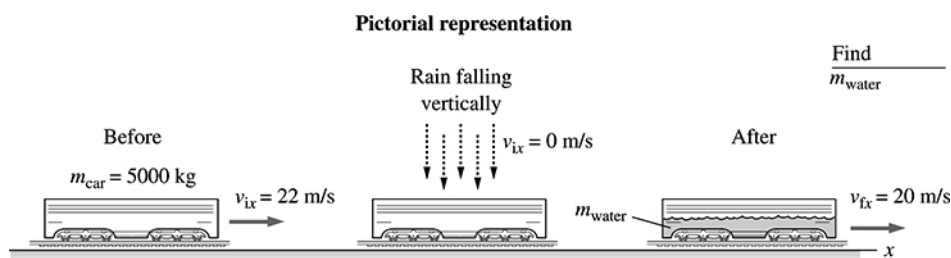


**Solve:** There are no *external* forces on the car + gravel system, so the horizontal momentum is conserved. This means  $p_{fx} = p_{ix}$ . Hence,

$$(10,000 \text{ kg} + 4000 \text{ kg})v_{fx} = (10,000 \text{ kg})(2.0 \text{ m/s}) + (4000 \text{ kg})(0.0 \text{ m/s}) \Rightarrow v_{fx} = 1.43 \text{ m/s}$$

**9.15. Model:** Choose car + rainwater to be the system.

**Visualize:**



There are no *external* horizontal forces on the car + water system, so the horizontal momentum is conserved.

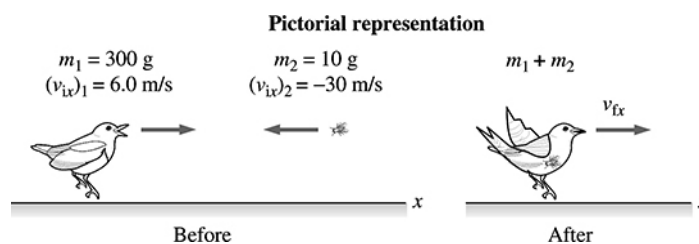
**Solve:** Conservation of momentum is  $p_{fx} = p_{ix}$ . Hence,

$$(m_{\text{car}} + m_{\text{water}})(20 \text{ m/s}) = (m_{\text{car}})(22 \text{ m/s}) + (m_{\text{water}})(0 \text{ m/s})$$

$$\Rightarrow (5000 \text{ kg} + m_{\text{water}})(20 \text{ m/s}) = (5000 \text{ kg})(22 \text{ m/s}) \Rightarrow m_{\text{water}} = 5.0 \times 10^2 \text{ kg}$$

**9.17. Model:** We will define our system to be bird + bug. This is the case of an inelastic collision because the bird and bug move together after the collision. Horizontal momentum is conserved because there are no external forces acting on the system during the collision in the impulse approximation.

**Visualize:**



**Solve:** The conservation of momentum equation  $p_{fx} = p_{ix}$  is

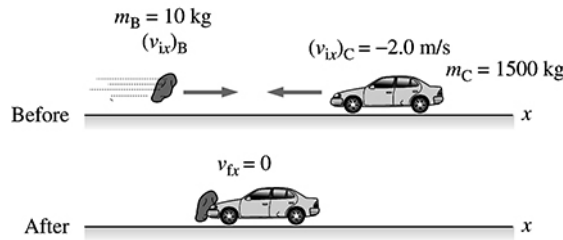
$$(m_1 + m_2)v_{fx} = m_1(v_{ix})_1 + m_2(v_{ix})_2 \Rightarrow (300 \text{ g} + 10 \text{ g})v_{fx} = (300 \text{ g})(6.0 \text{ m/s}) + (10 \text{ g})(-30 \text{ m/s}) \Rightarrow v_{fx} = 4.8 \text{ m/s}$$

**Assess:** We left masses in grams, rather than convert to kilograms, because the mass units cancel out from both sides of the equation. Note that  $(v_{ix})_2$  is negative.

**9.19. Model:** Because of external friction and drag forces, the car and the blob of sticky clay are not exactly an isolated system. But during the collision, friction and drag are not going to be significant. The momentum of the system will be conserved in the collision, within the impulse approximation.

**Visualize:**

**Pictorial representation**



**Solve:** The conservation of momentum equation  $p_{ix} = p_{ix}$  is

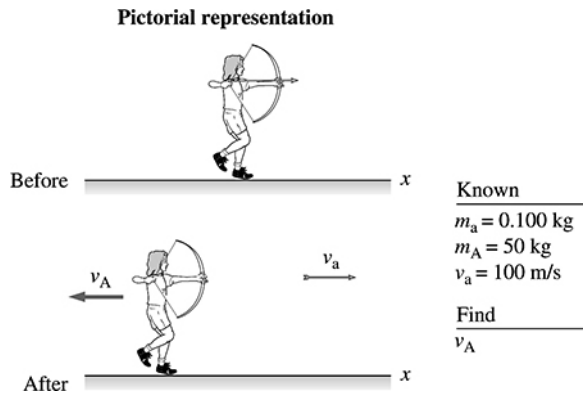
$$(m_C + m_B)(v_f)_x = m_B(v_{ix})_B + m_C(v_{ix})_C$$

$$\Rightarrow 0 \text{ kg m/s} = (10 \text{ kg})(v_{ix})_B + (1500 \text{ kg})(-2.0 \text{ m/s}) \Rightarrow (v_{ix})_B = 3.0 \times 10^2 \text{ m/s}$$

**Assess:** This speed of the blob is around 600 mph, which is very large. However, we must point out that a very large speed is *expected* in order to stop a car with only 10 kg of clay.

**9.20. Model:** We will define our system to be archer + arrow. The force of the archer (A) on the arrow (a) is equal to the force of the arrow on the archer. These are internal forces within the system. The archer is standing on frictionless ice, and the normal force by ice on the system balances the weight force. Thus  $\vec{F}_{ext} = \vec{0}$  on the system, and momentum is conserved.

**Visualize:**



The initial momentum  $p_{ix}$  of the system is zero, because the archer and the arrow are at rest. The final moment  $p_{ix}$  must also be zero.

**Solve:** We have  $M_A v_A + m_a v_a = 0 \text{ kg m/s}$ . Therefore,

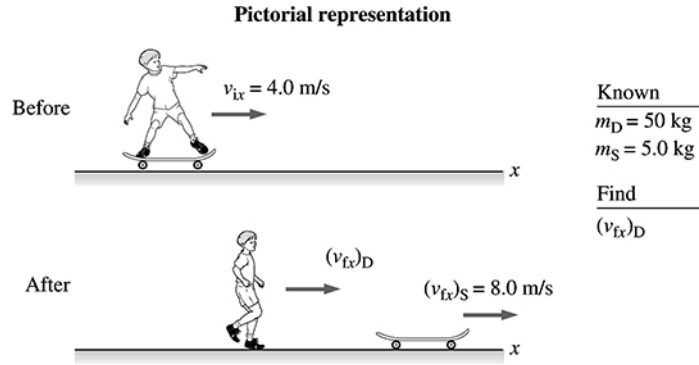
$$v_A = \frac{-m_a v_a}{m_A} = \frac{-(0.100 \text{ kg})(100 \text{ m/s})}{50 \text{ kg}} = -0.20 \text{ m/s}$$

The archer's recoil *speed* is 0.20 m/s.

**Assess:** It is the total final momentum that is zero, although the individual momenta are nonzero. Since the arrow has forward momentum, the archer will have backward momentum.

**9.22. Model:** We will define our system to be Dan + skateboard, and their interaction as an explosion. While friction is present between the skateboard and the ground, it is negligible in the impulse approximation.

**Visualize:**



The system has nonzero initial momentum  $p_{ix}$ . As Dan (D) jumps backward off the gliding skateboard (S), the skateboard will move forward in such a way that the final total momentum of the system  $p_{fx}$  is equal to  $p_{ix}$ .

**Solve:** We have  $m_S(v_{fx})_S + m_D(v_{fx})_D = (m_S + m_D)v_{ix}$ . Hence,

$$(5.0 \text{ kg})(8.0 \text{ m/s}) + (50 \text{ kg})(v_{fx})_D = (5.0 \text{ kg} + 50 \text{ kg})(4.0 \text{ m/s}) \Rightarrow (v_{fx})_D = 3.6 \text{ m/s}$$

**9.23. Model:** We assume that the momentum is conserved in the collision.

**Visualize:** Please refer to Figure EX9.23.

**Solve:** The conservation of momentum equation yields

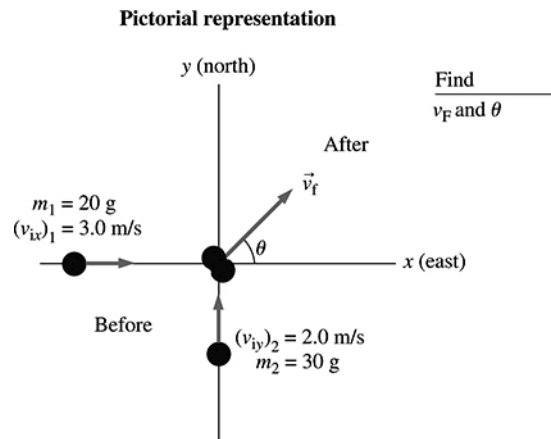
$$(p_{fx})_1 + (p_{fx})_2 = (p_{ix})_1 + (p_{ix})_2 \Rightarrow (p_{fx})_1 + 0 \text{ kg m/s} = 2 \text{ kg m/s} - 4 \text{ kg m/s} \Rightarrow (p_{fx})_1 = -2 \text{ kg m/s}$$

$$(p_{fy})_1 + (p_{fy})_2 = (p_{iy})_1 + (p_{iy})_2 \Rightarrow (p_{fy})_1 - 1 \text{ kg m/s} = 2 \text{ kg m/s} + 1 \text{ kg m/s} \Rightarrow (p_{fy})_1 = 4 \text{ kg m/s}$$

Thus, the final momentum of particle 1 is  $(-2\hat{i} + 4\hat{j}) \text{ kg m/s}$ .

**9.24. Model:** This problem deals with the conservation of momentum in two dimensions in an inelastic collision.

**Visualize:**



**Solve:** The conservation of momentum equation  $\vec{p}_{\text{before}} = \vec{p}_{\text{after}}$  is

$$m_1(v_{ix})_1 + m_2(v_{ix})_2 = (m_1 + m_2)v_{fx} \quad m_1(v_{iy})_1 + m_2(v_{iy})_2 = (m_1 + m_2)v_{fy}$$

Substituting in the given values,

$$(0.02 \text{ kg})(3.0 \text{ m/s}) + 0 \text{ kg m/s} = (0.02 \text{ kg} + 0.03 \text{ kg})v_f \cos \theta$$

$$0 \text{ kg m/s} + (0.03 \text{ kg})(2.0 \text{ m/s}) = (0.02 \text{ kg} + 0.03 \text{ kg})v_f \sin \theta$$

$$\Rightarrow v_f \cos \theta = 1.2 \text{ m/s} \quad v_f \sin \theta = 1.2 \text{ m/s}$$

$$\Rightarrow v_f = \sqrt{(1.2 \text{ m/s})^2 + (1.2 \text{ m/s})^2} = 1.7 \text{ m/s} \quad \theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1}(1) = 45^\circ$$

The ball of clay moves  $45^\circ$  north of east at 1.7 m/s.

**9.26. Model:** Model the rocket as a particle, and use the impulse-momentum theorem. The only force acting on the rocket is due to its own thrust.

**Visualize:** Please refer to Figure P9.26.

**Solve:** (a) The impulse is

$$J_x = \int F_x dt = \text{area of the } F_x(t) \text{ graph between } t = 0 \text{ s and } t = 30 \text{ s} = \frac{1}{2}(1000 \text{ N})(30 \text{ s}) = 1.5 \times 10^4 \text{ N s}$$

(b) From the impulse-momentum theorem,  $p_{fx} = p_{ix} + 1.5 \times 10^4 \text{ N s}$ . That is, the momentum or velocity increases as long as  $J_x$  increases. When  $J_x$  increases no more, the speed will be a maximum. This happens at  $t = 30 \text{ s}$ . At this time,

$$mv_{fx} = mv_{ix} + 1.5 \times 10^4 \text{ N s} \Rightarrow (425 \text{ kg})v_{fx} = (425 \text{ kg})(75 \text{ m/s}) + 1.5 \times 10^4 \text{ N s} \Rightarrow v_{fx} = 110 \text{ m/s}$$

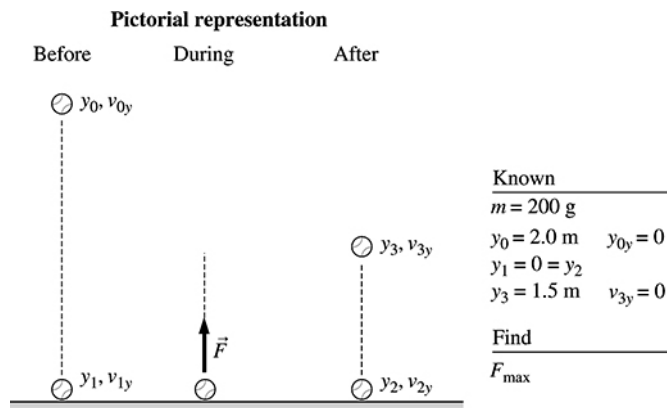
**9.27. Solve:** Using the equation

$$\begin{aligned} v_{fx} &= v_{ix} + \frac{\text{area under the force curve}}{m} \\ &= 0 \text{ m/s} + \frac{1}{0.250 \text{ kg}} \int_0^{2.0 \text{ s}} (10 \text{ N}) \sin\left(\frac{2\pi t}{4.0 \text{ s}}\right) dt \\ &= (40 \text{ N/m}) \left(\frac{4.0 \text{ s}}{2\pi}\right) \left(-\cos\left(\frac{2\pi t}{4.0 \text{ s}}\right)\right) \Big|_0^{2.0 \text{ s}} \\ &= -\left(\frac{80}{\pi} \text{ m/s}\right) (\cos(\pi) - \cos(0)) \\ &= 25 \text{ m/s} \end{aligned}$$

**Assess:** The force is applied for half the period of 4.0 s. During that time,  $\sin\left(\frac{2\pi t}{4.0 \text{ s}}\right)$  is positive, so an object initially at rest acquires a positive velocity.

**9.29. Model:** Model the ball as a particle that is subjected to an impulse when it is in contact with the floor. We will also use constant-acceleration kinematic equations. Ignore any forces other than the interaction between the floor and the ball during the collision in the impulse approximation.

**Visualize:**



**Solve:** To find the ball's velocity just before and after it hits the floor:

$$v_{1y}^2 = v_{0y}^2 + 2a_y (y_1 - y_0) = 0 \text{ m}^2/\text{s}^2 + 2(-9.8 \text{ m/s}^2)(0 - 2.0 \text{ m}) \Rightarrow v_{1y} = -6.261 \text{ m/s}$$

$$v_{3y}^2 = v_{2y}^2 + 2a_y (y_3 - y_2) \Rightarrow 0 \text{ m}^2/\text{s}^2 = v_{2y}^2 + 2(-9.8 \text{ m/s}^2)(1.5 \text{ m} - 0 \text{ m}) \Rightarrow v_{2y} = 5.422 \text{ m/s}$$

The force exerted by the floor on the ball can be found from the impulse-momentum theorem:

$$\begin{aligned} mv_{2y} &= mv_{1y} + \int F dt = mv_{1y} + \text{area under the force curve} \\ &\Rightarrow (0.200 \text{ kg})(5.422 \text{ m/s}) = -(0.200 \text{ kg})(6.261 \text{ m/s}) + \frac{1}{2} F_{\text{max}} (5.0 \times 10^{-3} \text{ s}) \end{aligned}$$



$$\Rightarrow F_{\max} = 9.3 \times 10^2 \text{ N}$$

**Assess:** A maximum force of  $9.3 \times 10^2 \text{ N}$  exerted by the floor is reasonable.

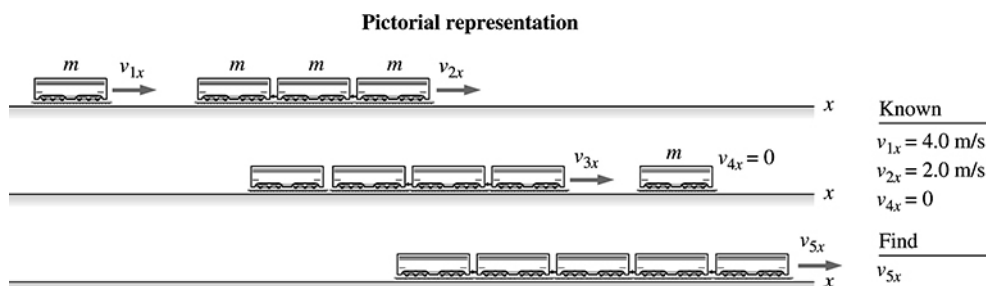
**9.32. Solve:** Using Newton's second law for the  $x$ -direction,  $F_x = \frac{dp_x}{dt}$ . Therefore,

$$F_x = \frac{d}{dt}(6t^2 \text{ kg m/s}) = 12t \text{ N}$$

**Assess:** The  $x$ -component of the net force on an object is equal to the time rate of change of the  $x$ -component of the object's momentum.

**9.34. Model:** Model the train cars as particles. Since the train cars stick together, we are dealing with perfectly inelastic collisions. Momentum is conserved in the collisions of this problem in the impulse approximation, in which we ignore external forces during the time of the collision.

**Visualize:**



**Solve:** In the collision between the three-car train and the single car:

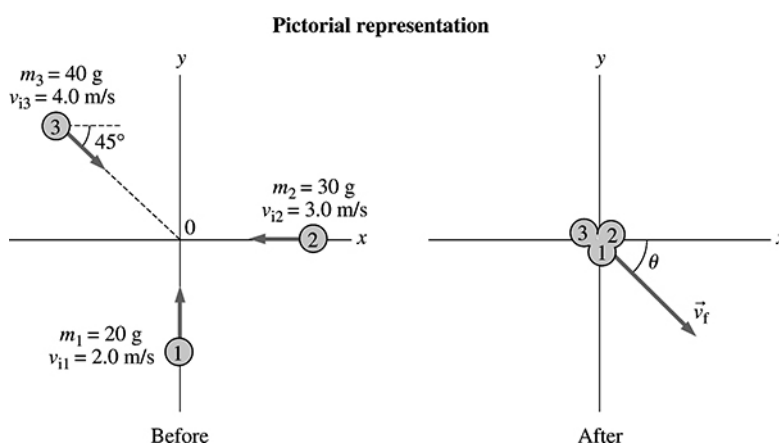
$$mv_{1x} + (3m)v_{2x} = 4mv_{3x} \Rightarrow v_{1x} + 3v_{2x} = 4v_{3x} \Rightarrow (4.0 \text{ m/s}) + 3(2.0 \text{ m/s}) = 4v_{3x} \Rightarrow v_{3x} = 2.5 \text{ m/s}$$

In the collision between the four-car train and the stationary car:

$$(4m)v_{3x} + m v_{4x} = (5m)v_{5x} \Rightarrow 4v_{3x} + 0 \text{ m/s} = 5v_{5x} \Rightarrow v_{5x} = \frac{4v_{3x}}{5} = (0.8)(2.5 \text{ m/s}) = 2.0 \text{ m/s}$$

**9.59. Model:** Model the three balls of clay as particle 1 (moving north), particle 2 (moving west), and particle 3 (moving southeast). The three stick together during their collision, which is perfectly inelastic. The momentum of the system is conserved.

**Visualize:**



**Solve:** The three initial momenta are

$$\vec{p}_{11} = m_1 \vec{v}_{11} = (0.020 \text{ kg})(2.0 \text{ m/s}) \hat{j} = 0.040 \hat{j} \text{ kg m/s}$$

$$\vec{p}_{12} = m_2 \vec{v}_{12} = (0.030 \text{ kg})(-3.0 \text{ m/s} \hat{i}) = -0.090 \hat{i} \text{ kg m/s}$$

$$\vec{p}_{13} = m_3 \vec{v}_{13} = (0.040 \text{ kg})[(4.0 \text{ m/s}) \cos 45^\circ \hat{i} - (4.0 \text{ m/s}) \sin 45^\circ \hat{j}] = (0.113 \hat{i} - 0.113 \hat{j}) \text{ kg m/s}$$

Since  $\vec{p}_f = \vec{p}_i = \vec{p}_{i1} + \vec{p}_{i2} + \vec{p}_{i3}$ , we have

$$(m_1 + m_2 + m_3)\vec{v}_f = (0.023\hat{i} - 0.073\hat{j}) \text{ kg m/s} \Rightarrow \vec{v}_f = (0.256\hat{i} - 0.811\hat{j}) \text{ m/s}$$

$$\Rightarrow v_f = \sqrt{(0.256 \text{ m/s})^2 + (-0.811 \text{ m/s})^2} = 0.85 \text{ m/s}$$

$$\theta = \tan^{-1} \frac{|v_{fy}|}{v_{fx}} = \tan^{-1} \frac{0.811}{0.256} = 72^\circ \text{ below } +x$$